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reject the previous optimal solution, hoping to increase the value of the bound; they are computed using a mixed-integer linear program (using one binary variable per region in $\hat{O}_K$). We stop when the (rounded) bound is equal the true nonnegative rank, or when no new direction can be found.

Results are displayed on the fourth row of Table 1. This dynamic approach appears to require more computational effort. Nevertheless, we observe for 4-, 5- and 6-gons that it finds lower bounds that are as good as the fixed $\hat{\mathcal{D}}_6$ bound, but with much fewer directions (for instance 46 directions instead of more than five thousands for the 6-gon). In essence this provides us with more compact certificates for those nonnegative ranks. The case of the dynamic approach for the 7-gon is slightly different: it was started with the full set of directions $\hat{\mathcal{D}}_6$, which gives a bound equal to 3.5, and the goal was to improve that bound. Instead, the procedure found that no direction can improve the bound, implying that improving the bound requires to increase the outer discretization parameter K. This seems to support our claim that the higher the value of n, the finer the discretization is needed to be to obtain good lower bounds.

4 Conclusion

In this paper, we introduced new lower bounds on the nonnegative rank based on a nested polytopes formulation. Numerical experiments on small slack matrices demonstrate that this approach is promising, as the bound can be close to the true nonnegative rank or even tight. In the future, we plan to focus on the computational efficiency of our procedure, which will allow tests on larger matrices with higher values of the discretization parameters N and K.

References

[1] Stephen A. Vavasis. On the complexity of nonnegative matrix factorization. *SIAM Journal on Optimization*, 20(3):1364–1377, 2009.

[2] Mihalis Yannakakis. Expressing combinatorial optimization problems by linear programs. *Journal of computer and system sciences*, 43:441–466, 1991.

[3] Hamza Fawzi and Pablo A Parrilo. Lower bounds on nonnegative rank via nonnegative nuclear norms. *Mathematical Programming*, 153(1):41–66, 2015.

[4] Hamza Fawzi and Pablo A Parrilo. Self-scaled bounds for atomic cone ranks: applications to nonnegative rank and cp-rank. *Mathematical Programming*, 158(1-2):417–465, 2016.

[5] Samuel Fiorini, Volker Kaibel, Kanstantsin Pashkovich, and Dirk Oliver Theis. Combinatorial bounds on nonnegative rank and extended formulations. *Discrete Mathematics*, 313(1):67–83, 2013.

[6] Nicolas Gillis and François Glineur. On the geometric interpretation of the nonnegative rank. *Linear Algebra and its Applications*, 437(11):2685–2712, 2012.

[7] Joel E. Cohen and Uriel G. Rothblum. Nonnegative ranks, decompositions, and factorizations of nonnegative matrices. *Linear Algebra and its Applications*, 190:149–168, 1993.

[8] Alok Aggarwal, Heather Booth, Joseph O'Rourke, Suri Surhash, and Chee K. Yap. Finding minimal convex nested polygons. *Information and Computation*, 83(1):98–110, 1989.

[9] Michael G Dobbins, Andreas Holmsen, and Tillmann Miltzow. A universality theorem for nested polytopes. *arXiv preprint arXiv:1908.02213*, 2019.