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A mental model for the solution of the direct and inverse kinematic problem

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Abstract. A model is proposed which is based on a recurrent network, representing the geometric properties of the human arm. The direct and the inverse kinematic problem as well as any mixed problem, including the underconstrained case, can be solved by the network. It is realized for the example of a three-joint manipulator working in a two-dimensional plane, i.e. for a manipulator with one extra degree of freedom.

1.Introduction

An important problem of motor control is the coordination of movement of a multisegment manipulator, as for example the human arm. A number of solutions exist for the direct kinematic problem, which means to compute work space coordinates $\mathbf{x}$ from joint space coordinates $\mathbf{q}$, and the inverse kinematic problem (computation of $\mathbf{q}$ from $\mathbf{x}$), but these solutions have several drawbacks. First, usually a supervising system is necessary to decide which computation is required for the particular task. Second, the solutions require a number of calculations, in particular, when the manipulator is redundant. Third, when calculating the inverse kinematic problem, the appearance of singularities enforce numerical unstable solutions. The system we propose here does not involve these problems.

Mussa Ivaldi, Morasso, Zaccaria (1988) pointed out that the inverse kinematic problem could easily be solved by a mechanical model of the manipulator, provided with springs to simulate the muscles. They called this the Passive Motion Paradigm. A similar idea was proposed by Hinton (1984). However, how can the properties of such a mechanical model be implemented in a neural network to form the basis of a "mental model" of the arm? Here we propose such a system which represents the geometry of the multisegment arm.

2.Theory

We consider a manipulator with n degrees of freedom. These might be provided by rotational joints or, if the length of the limbs can be changed, by translational joints. The system is redundant when, given an m-dimensional work space, n is larger than m. The state of the system can be described by at least $n + m$ variables although it is uniquely defined by any subset of n out of these $n + m$ state variables. The task of the system proposed here is to complete the full set of $n + m$ values, given any subset of n values. We will also show that a geometrically possible, "true" state can be constructed even if less than n values are given to the system.

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Principally, the property of reconstructing a set of state values is known from the classical Hopfield network (1984). The complete set can be considered as a point-attractor. However, the Hopfield net is not really appropriate for our task because it permits only a limited number of attractors whereas, within the geometrically possible borderlines, every point of the n-dimensional subspace of the $(n + m)$-dimensional state space should be able to act as an attractor. Therefore the following quantitative and qualitative changes to the basic Hopfield structure are introduced. The nonlinear properties of the activation functions, which in principle also occur in the Hopfield net, show a larger variation. As a further nonlinearity, multiplicative interactions between the outputs of two neurons are possible. A major qualitative difference is that in our system the connections between two neurons are allowed to be strongly asymmetrical. Apart from these structural differences, the most significant property of our system is the following. A strong redundancy is introduced into the system as the value of one state variable is calculated not only once, but independently in several different ways. The final value of this variable is then determined by calculating a mean value. This will be summarized here as the Mean of Multiple Computations *(MMC)*.

2.1 Simulations

A realization of a redundant system is presented here with $n = 3$ and $m = 2$ (Fig 1a,b). In addition to the joint angles, precursors which are defined parts of a joint angle are investigated. This leads to 17 internal state variables. Each corresponds to an output unit in the network (the units in the horizontal row in Fig 2). The multiple calculations of a variable will be explained using the angle β as an example. The following equations are used to calculate β:

$$\beta = arccos\left(\frac{L_1^2 + L_2^2 - D_2^2}{2L_1L_2}\right) \tag{1}$$

$$\beta = \beta_1 + \beta_2 \tag{2}$$

$$\beta = \pi - (\epsilon_1 + \gamma_1) \tag{3}$$

$$\beta = \pi - (\epsilon - \epsilon_2 + \gamma - \gamma_2) \tag{4}$$

$$\beta = \pi - (\epsilon_1 + \gamma - \gamma_2) \tag{5}$$

$$\beta = \pi - (\epsilon - \epsilon_2 + \gamma_1) \tag{6}$$

In Fig 2 each set of equations is symbolized by a black rectangle. The number of equations used for each variable is symbolized by the number of arrows connecting the black rectangle and the circular unit below. With the appropriate hardware the calculations can all be performed in parallel and thus very fast.

The multiple computation of the internal variables introduces a redundancy into the system which has to be removed in order to obtain unique values. This is done by calculating a mean value of the different solutions. Several possibilities exist. Here we used the simple arithmetic mean. The output of these mean

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calculations, which are symbolized in Fig 2 as circles, are fed back to the input layer (the left-hand column in Fig 2) and are used for the calculation of the next cycle. A value can arrive at the input layer either via the feedback line, or it can be provided as an external input value. The internal feedback channel is suppressed when an external signal appears.

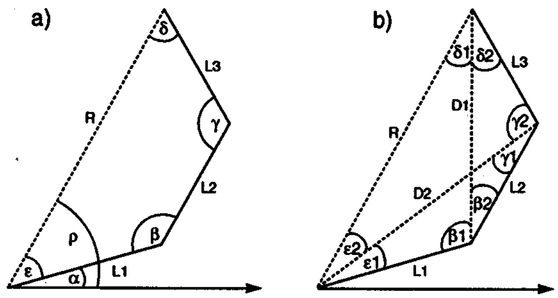

Fig.1. Definition of the geometric values to describe the position of a three-joint manipulator working in a two-dimensional plane.

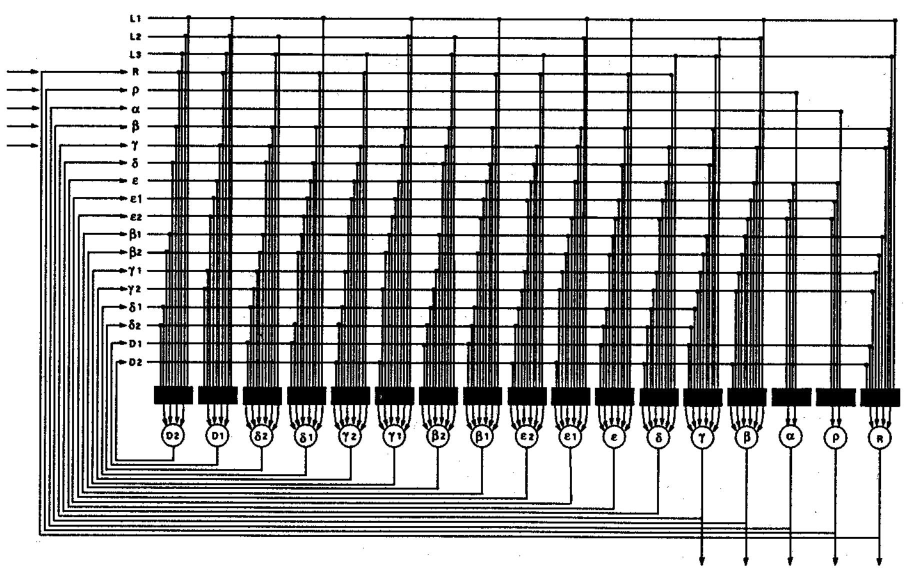

Fig.2. Principal structure of the network used to model the geometric properties of the three-joint arm shown in Fig 1.

2.2 Results

The system is experimentally tested for the following situations: Nine target points, numbered 1 to 9, are selected, which are distributed over a large part of the work space (Fig 3a). For each endpoint an arbitrary arm position was chosen which was then defined by the values of the joint angles α, β, and γ

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shown in Fig 3. The relaxation behavior for the solution of the direct kinematic problem is shown in Fig 4d,e. Fig 4f shows the geometric distance between the tip of the arm, defined by the endpoint coordinates (R, ρ) and the target point (target error). The target is considered to be reached when the error is smaller than 0.01 length units. In all cases investigated the system solves the problem of direct kinematics. When using another set of angle values for these 9 target points, the behavior was not found to be essentially different.

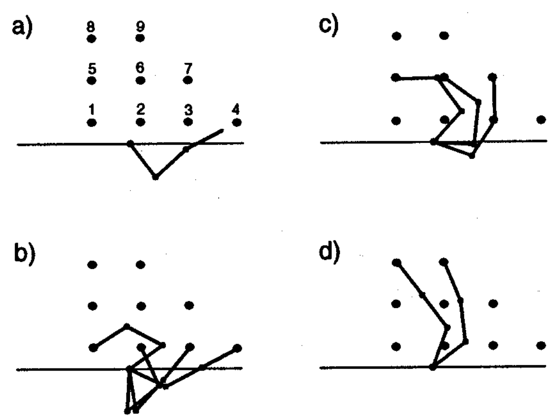

Fig.3. (a) Arrangement of the nine target points and starting position of the arm. (b) - (d) Positions of the arm when the target points are reached.

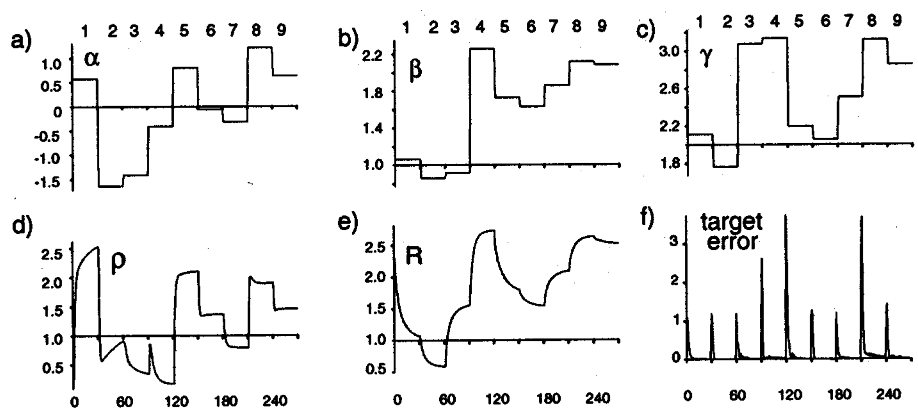

Fig.4. Direct kinematics. (a) - (c) given angle values for the 9 target points, each over 30 iteration cycles. (d) - (e) Iteration of coordinates ρ and R.

To test the system when solving the inverse kinematic problem, the same target points and the same starting position as for Figs 3 were used. In this case only the coordinates of the target points are given as input. This means that the system is underconstrained. Fig 5 shows the behavior in the same way as was done above for the direct kinematics. In Fig 5f the target error shows

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the distance between the target point and the endpoint coordinates of the arm, calculated from the actual values of α, β and γ. Again in most cases the target is reached in less than 30 time steps. In one case (point 4) the system needed about 60 steps to converge.

In the example of Fig 5 the arm started in a somewhat extreme position as the angle of the hand joint has a negative value (see Fig 3a). When starting from a more "comfortable" position the system relaxes to different positions for each target point and the relaxation is a little faster in some cases.

Fig 6 shows an example of a mixed problem. The target coordinate ρ plus two other angles, β and γ, are used as input. In this case the relaxation needs a little more time for the points 1 - 3, but relatively less time for point 4.

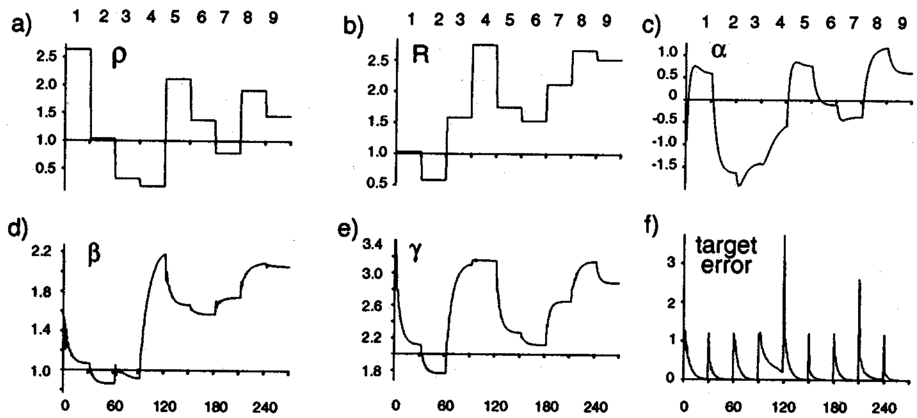

Fig.5. Inverse kinematics, iteration of joint angle values α, β and γ

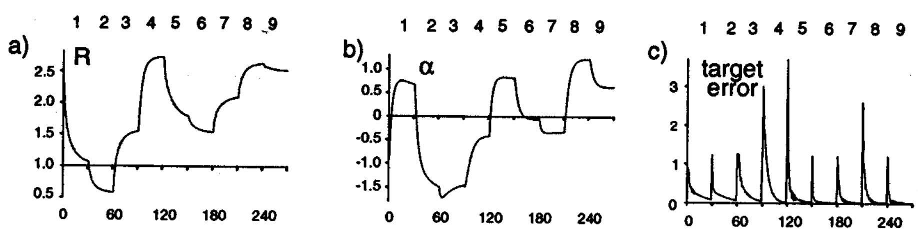

Fig 6 Solving a mixed problem. As input to the system one polar coordinate (ρ) and two joint angles, β and γ, are used. These values can be found in Fig 5a, 4b, and 4c, respectively.

2.3 Discussion

The *MMC* system shown here is based on a type of network which can be applied when (i) the variables of the system depend on each other, (ii) these dependencies can be described quantitatively and (iii) more variables can be defined than are necessary to describe the state of the system. In our experiments we never found the network not to converge. Work to prove the general convergence properties are in progress.

Two possibilities exist to use the system for direct control of a robot arm. Either the output values have to be frozen until the internal system has relaxed. As an alternative, the actual values for α, β, and γ are used to control the arm. Oscillations occuring in these values might be overcome by using either a low pass filter or, more simply, only the output values of every second time step.

The occurrence of singularities would mean that one of the different parallel computations provides no significant value. Due to the redundancy of the network this does not significantly affect the behavior of the whole system. Generally, the highly parallel structure makes the system rather independent of errors ocurring within the system.

Morasso's model (1989, 1991) directly provides a signal to control the muscles, whereas our model only provides angle values. On the other hand, in our system we can easily fix some arbitrary variables, and thus can also solve the direct kinematic problem. Our system works on a higher level of abstraction and, therefore, opens the possibility to introduce different strategies to control the redundancy, such as, for example, the minimum cost principle (Cruse 1986).

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