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Trajectory Learning Using Hierarchy of Oscillatory Modules

Nikzad Benny Toomarian and Pierre Baldi *

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, CA 91109

Abstract. To this date, the most successful approaches to learning have been either the back-propagation or gradient descent method. Although very powerful on relatively simple problems, theoretical analysis and simulations show that these approaches break down as soon as sufficiently complex problems are considered. To overcome this fundamental limitation, we suggest a hierarchical and modular approach, directly inspired from biological networks, whereby a certain degree of structure is introduced in the learning system. This approach is applied to a simple example of trajectory learning of a semi-figure eight. The ideas involved, however, extend immediately to more general computational problems.

1. Introduction

Learning is a fundamental ability of biological systems. Understanding its principles is also key to the design of intelligent circuits and computers. To this date, the most successful approach to learning, from an engineering standpoint, has been the back-propagation approach[7] or gradient descent approach. In this framework, in the course of learning from examples, the parameters of a learning system, such as a neural network, are adjusted incrementally so as to optimize by gradient descent a suitable function measuring the performance of the system at any given time. Although very powerful on relatively simple problems, theoretical analysis and simulations[3,4] show that this approach breaks down as soon as sufficiently complex problems are considered. Gradient descent learning applied to an amorphous learning system is bound to fail. To overcome this fundamental limitation, we are suggesting a hierarchical and modular approach whereby a certain degree of structure is introduced in the learning system.

Consider the problem of synthesizing a neural network capable of producing a certain given non-trivial trajectory. To fix the ideas, we can imagine that the model neurons in the network satisfy the usual additive model equations[5]

$$\frac{du_i}{dt} = -\frac{u_i}{\tau_i} + \sum_j w_{ij} f(u_j) + I_i \qquad (1)$$

The learning task is to find the right parameter values, such as the synaptic weights w_{ij}, the charging time constants τ_i and the amplifiers gains, so that the output units of the network follow a certain prescribed trajectory $u^*(t)$ over a given time interval $[t_0, t_1]$. For instance, a typical benchmark trajectory in the literature is a circle or a figure eight. Networks such as (1) have been successfully

* and Division of Biology, California Institute of Technology.

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trained on figure eights using a form of gradient descent learning for recurrent networks[6,8]. Consider now the problem of learning a more complicated trajectory, such as a double figure eight. Although the task appears only slightly more complicated, simulations show that a fully interconnected set of units will not be able to learn this task by indiscriminate gradient descent learning on all of its parameters. Thus a different approach is needed.

2. Modular Hierarchical Approach

Biology seems to have overcome the obstacles inherent to gradient descent learning through evolution. Learning in biological organisms is never started from a tabula rasa. Rather, a high degree of structure is already present in the neural circuitry of newly born organisms. This structure is genetically encoded and the result of evolutionary tinkering over time scales several times larger than those of continental drift. Little is known of the interaction between the prewired structure and the actual learning. One reasonable hypothesis is that complex tasks are broken up into simpler modules and that learning, perhaps in different forms, can operate both within and across modules. The modules in turn can be organized in a hierarchical way, all the way up to the level of nuclei or brain areas. The difficult problem then becomes how to find a suitable module decomposition and whether there are any principles for doing so. One trick used by evolution seems to have been the duplication, by error, of a module together with the subsequent evolution of one of the copies into a new module somehow complementary of the first one. But this is far from yielding any useful principle and may, at best, be used in genetic type of algorithms, where evolutionary tinkering is mimicked in the computer.

We have taken inspiration from these ideas, to tackle the problem of learning specific complex trajectories in a neural network. Although it is difficult at this stage to keep a close analogy with biology, it may be useful to think of the problem of central pattern generation or motor control in natural organisms. In order to construct a neural network capable of producing a double figure eight, we are going to introduce a certain degree of organization in the system prior to any learning. The basic organization of the system consists of a hierarchy of modules. In this particular example, each module can be viewed essentially as an oscillator. The modules, in turn, are organized in a hierarchical way. For the time being, all the modules within one level of the hierarchy control the output of the modules located in the previous layer.

At the bottom of the hierarchy, in the first level, one finds a family of simple and possibly independent modules, each one corresponding to a circuit with a small number of units capable of producing some elementary trajectory, such as a sinusoidal oscillation. In the case of the additive model, these could be simple oscillator rings with two or three neurons, an odd number of inhibitory connections and sufficiently high gains[1,2]. Thus, in our example, the first level of the hierarchy could contain four oscillator rings, one for each loop of the target trajectory. The parameters in each one of these four modules can be adjusted, e.g., by gradient descent, in order to match each one of the loops in the target trajectory.

The second level of the pyramid should contain two control modules. Each one of these modules controls a distinct pair of oscillator networks from the first level, so that each control network in the second level ends up producing a simple figure eight (see Fig. 1). Again, the control networks in level two can

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be oscillator rings and their parameters can be adjusted. In particular, after the learning process is completed, they should be operating in their high-gain regimes and have a period equal to the sum of the periods of the circuits each one controls.

Finally, the third layer, consist of another oscillatory and adjustable module which controls the two modules in the second level so as to produce a double figure eight. The third layer module must also end up operating in its high-gain regime. In general, the final output trajectory is also a limit cycle because it is obtained by superimposition of limit cycles in the various modules. If the various oscillators relax to their limit cycles independently of one another, it is essential to provide for adjustable delays between the various modules in order to get the proper harmony among the various phases. In this way, a sparse network with 20 units or so can be constructed which can successfully execute a double figure eight.

It is clear that this approach which combines a modular hierarchical architecture together with some simple form of learning can be extended to general trajectories. At the very least, one could always use Fourier analysis to decompose a target trajectory into a superimposition of sinusoidal oscillations of different frequencies and use, in the first level of the hierarchy, a corresponding large bank of oscillators networks. One could also use damped oscillators to perform some sort of wavelet decomposition. Although we believe that oscillators with limit cycles present several attractive properties (stability, short transients, biological relevance...), one can conceivably use completely different circuits as building blocks in each module.

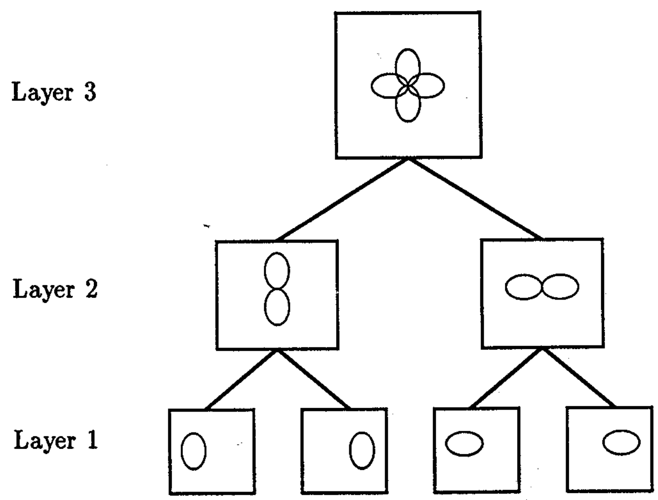

Fig. 1: Symbolic representation of a modular and hierarchical network for double figure eight.

The modular hierarchical approach leads to architectures which are more structured than fully interconnected networks, with a general feedforward flow of information and sparse recurrent connections to achieve dynamical effects. The sparsity of units and connections are attractive features for hardware design;

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and so is also the modular organization and the fact that learning is much more circumscribed than in fully interconnected systems. However, fundamental open problems remain in the overall organization of learning across modules and in the origin of the decomposition. In particular, can the modular architecture be the outcome of a simple internal organizational process rather than an external imposition and how should learning be coordinated in time and across modules (other than the obvious: modules in the first level learn first, modules in the second level second,...)? How successful is a global gradient descent strategy applied across modules? How can the same modular architecture be used for different trajectories, with short switching times between trajectories and proper phases along each trajectory?

3. Example of Numerical Simulations

The new learning paradigm, presented in the preceding section, has been applied to the problem of learning a figure eight trajectory. Results referring to this problem can be found in the literature[6,8].

In this work we assumed that the desired trajectory of a semi-figure eight is composed of two circles and given by:

$$D_1 = C_1\ [x_{10} + cos(t)]\ +\ (1 - C_1)[y_{10} - cos(t)] \tag{2a}$$

$$D_2 = C_1\ [x_{20} + sin(t)]\ +\ (1 - C_1)[y_{20} + sin(t)] \tag{2b}$$

in which C_1 is a square wave with a period of 4π, given by the following equation;

$$C_1 = sign[sin(t/2)] \tag{3}$$

and $x_{10}, x_{20}, y_{10}, y_{20}$ are the coordinates of the center of the left and right circles respectively. Plotting D_1 vs. D_2 will produce the desired semi-figure eight, as shown in fig. 3.

The basic module of the hierarchical approach for this trajectory is a simple oscillatory ring network with four neurons. The activation dynamics of each unit in the module is given by:

$$\frac{du_i}{dt} = -\frac{u_i}{\tau_i} + w_{i-1}V_{i-1} \quad i = 1, \cdots, 4 \tag{4}$$

where $V_0 = V_4$ and V_i is the output of neuron i given by;

$$V_i = tanh(\gamma_i\ u_i) \tag{5}$$

An odd number of inhibitory connections is required for stable oscillations (Atiya and Baldi 1989). At this stage for simplicity, we assume that $w_i = w$ for $i = 1, 3, 4$, $w_2 = -w$ and $\tau_i = \tau, \gamma_i = \gamma$ for $i = 1, \cdots, 4$. The module is trained to produce a circle through a sinusoidal waive with period of 2π. Following the analysis in Atiya and Baldi 1989, the initial value of the network parameters, i.e., w, τ and γ are set to one at the beginning of the learning procedure. To update the network parameters, a gradient descent algorithm based upon the forward propagation of the error is used[9]. After the training, the network parameters

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converge to the following values, $w = 1.025, \tau = 0.972$ and $\gamma = 1.526$. With these values, after a brief transition period, the module converges to a limit cycle where each unit has a quasi-sinusoidal activation. The phase shift between two consecutive neurons is about $\pi/4$. Therefore, plotting the activity of neuron 1 and 3 in the module against each other will produce a circle which is close to the desire one as illustrated in Fig. 2.

At the second level of the hierarchy is the control module. This module is also chosen to be a simple oscillatory ring network with four neurons. This network is operating in the high gain regime and its period is twice that of the basic modules, i.e., 4π. The network parameters at the beginning of the learning are set to $w = 0.9, \gamma = 10$, and $\tau = 2.58$.

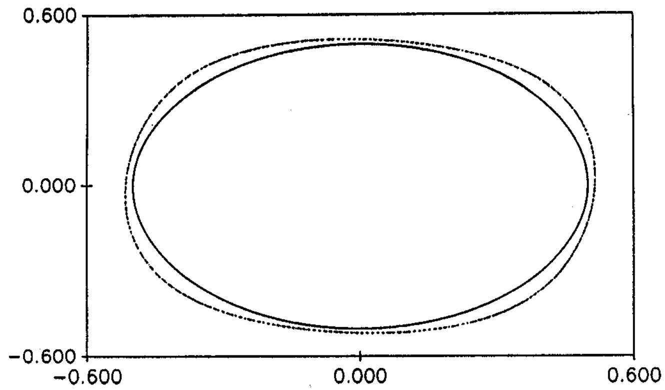

Fig. 2: Desired circle (solid line) and the one produced by the basic module in the first layer (dashed line).

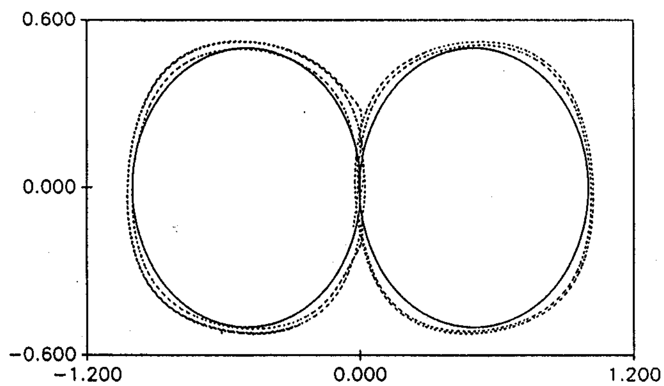

Fig. 3: Desired semi-figure eight (solid line) and the one produced by the network (dashed line).

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The overall network has two output at any time, Z_1 and Z_2. Their value is given by:

$$Z_1 = 0.5\{[1 + VC(1)]\ [x_{10} + VN1(1)]\ +\ [1 - VC(1)]\ [y_{10} + VN1(3)]\} \quad (6a)$$

$$Z_2 = 0.5\{[1 + VC(1)]\ [x_{20} + VN2(1)]\ +\ [1 - VC(1)]\ [y_{20} + VN2(3)]\} \quad (6b)$$

in which $VN1(i)$ and $VN2(i)$ are the output of i^{th} neuron in the first and second modules in the first level of the hierarchy, respectively, where $VC(1)$ is the output of the first neuron in the control module. Figure 4 shows the semi-figure eight obtained be plotting Z_1 vs. Z_2.

4. Conclusion

In conclusion, a new hierarchical approach for supervised neural learning of time dependent trajectories is presented. The modular hierarchical methodology leads to architectures which are more structured than fully interconnected networks, with a general feedforward flow of information and sparse recurrent connections to achieve dynamical effects. The sparsity of the connections as well as the modular organization makes the hardware implementation of the methodology very easy and attractive. This approach has been applied to an example of trajectory learning of a semi-figure eight.

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