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A Learning Algorithm for the Blind Separation of Non-zero Skewness Source Signals with No Spurious Equilibria

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Abstract. Neural computational approach to blind sources separation was first introduced by Jutten and Herault [6], and further developed by others [9, 3, 7, 4]. Necessary and sufficient conditions for the blind sources separation have been proposed by Cardoso [1], Tong *et al* [10, 11], and Common [5]. There have been difficulties of implementing necessary and sufficient conditions by a neural network with no spurious equilibria. In this paper, we present a necessary and sufficient condition for the blind sources separation, which can be implemented by a neural network with no spurious equilibria. Specifically, if the source signals are independent and each of them has a non-zero skewness (3rd-order cumulant), then the sources are separated by a linear transformation, if and only if all the 2nd- and 3rd-order *cross*-cumulants of the output are zero. This condition does not require the 3rd-order cumulants among three different variables to be zero. Because the condition requires only *pairwise* statistics (statistics between two different variables), it can be implemented by a neural network with no spurious equilibria.

1. Blind Sources Separation

Consider the case where the observation vector $\mathbf{x}(t) \in \mathbb{R}^n$ and the source vector $\mathbf{s}(t) \in \mathbb{R}^n$ are related by

$$\mathbf{x}(t) = A\mathbf{s}(t). \tag{1}$$

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The problem of blind sources separation is to recover the source signals $\mathbf{s}(t)$ from the observation vector $\mathbf{x}(t)$ without the knowledge of the mixing matrix A. In other words, it is required to find a linear transformation B, i.e.,

$$\mathbf{y}(t) = B\mathbf{x}(t), \tag{2}$$

such that the composite matrix $H = BA$ has the decomposition,

$$H = \Pi\Lambda, \tag{3}$$

for some permutation matrix Π and nonsingular diagonal matrix Λ.
Throughout this paper, the following assumptions hold:
A4.1: $A \in \mathbb{R}^{n\times n}$ is nonsingular.
A4.2: At each time, the components of $\mathbf{s}(t)$ are statistically independent.
A4.3: Each component of $\mathbf{s}(t)$ is a zero mean ergodic stationary process with a non-zero variance.
A4.4: Each component of $\mathbf{s}(t)$ has a non-zero skewness, i.e.,

$$< s_i^3(t) > \neq 0 \quad \text{for } i = 1, \ldots, n. \tag{4}$$

2. A Necessary and Sufficient Condition

The main theorem states a necessary and sufficient condition for the blind separation of source signals. Let R_s denote $< \mathbf{s}(t)\mathbf{s}^T(t) >$. Let C_s be a diagonal matrix whose ith diagonal element is $< s_i^3(t) >$, where $< \cdot >$ represent expectation operator.

Theorem 1 (Main Theorem) *Let the sources $\mathbf{s}(t)$ satisfy the assumptions given in A1 through A4. Then, H has decomposition (3), i.e.,*

$$H = \Pi\Lambda,$$

if and only if the following conditions are satisfied:

$$HR_sH^T = \Lambda_1, \tag{5}$$
$$HC_s(H \circ H)^T = \Lambda_2, \tag{6}$$

where Λ_1 and Λ_2 are diagonal matrices with non-zero diagonal entries. The operator $\circ$ denotes Hardamard product.

Proof: See Appendix.

For the proof of main theorem, the following lemma is necessary.

Lemma 1 *Let $Q \in \mathbb{R}^{n\times n}$ be an orthogonal matrix. Let $\Sigma_1 \in \mathbb{R}^{n\times n}$ and $\Sigma_2 \in \mathbb{R}^{n\times n}$ be diagonal matrices with non-zero diagonal entries. If $\Sigma_1(Q \circ Q) = Q\Sigma_2$, then Q has the decomposition, $Q = \Pi \overset{\circ}{I}$, where Π is some permutation matrix and $\overset{\circ}{I}$ is some diagonal matrix whose diagonal entries are either +1 or -1.*

Proof: See [2]

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3. A Learning Algorithm

Consider a feedforward network with lateral feedback connections as shown in Figure 1.

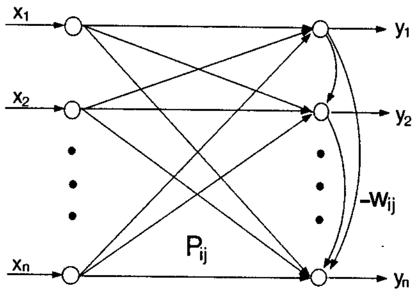

Figure 1: Schematic architecture of the network with n nodes

The output $\mathbf{y}(t)$ of the network is given by

$$\begin{aligned} \mathbf{y}(t) &= P(t)\mathbf{x}(t) - W(t)\mathbf{y}(t) \\ &= (I + W(t))^{-1}P(t)\mathbf{x}(t), \end{aligned} \tag{7}$$

where

$$P(t) = \begin{bmatrix} p_{11}(t) & p_{12}(t) & \cdots & p_{1n}(t) \\ p_{21}(t) & p_{22}(t) & \cdots & p_{2n}(t) \\ \vdots & & & \vdots \\ p_{n1}(t) & p_{n2}(t) & \cdots & p_{nn}(t) \end{bmatrix},$$
$$W(t) = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ w_{21}(t) & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ w_{n1}(t) & w_{n2}(t) & \cdots & 0 \end{bmatrix}. \tag{8}$$

The learning algorithm is given by

$$\frac{dP(t)}{dt} = \beta\{\Lambda_1 - \mathbf{y}(t)\mathbf{y}^T(t) + \mathbf{y}(t)\mathbf{f}^T(\mathbf{y}(t)) - \mathbf{f}(\mathbf{y}(t))\mathbf{y}^T(t)\}P(t), \tag{9}$$
$$\frac{dw_{ij}(t)}{dt} = \gamma\{y_i(t)y_j^2(t)\}, \quad \text{for } i > j, \tag{10}$$

where $\mathbf{f}(\mathbf{y}(t)) = \mathbf{y}(t) \circ \mathbf{y}(t) = [y_1^2(t), \ldots, y_n^2(t)]^T$, Λ_1 is a diagonal matrix whose diagonal elements are prespecified, and β, γ are learning rates, small positive constants. The convergence of (9) and (10) is achieved when the following equations are satisfied:

$$< \Lambda - \mathbf{y}(t)\mathbf{y}^T(t) + \mathbf{y}(t)\mathbf{f}^T(\mathbf{y}(t)) - \mathbf{f}(\mathbf{y}(t))\mathbf{y}^T(t) > \; = \; 0, \tag{11}$$

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and

$$< y_i(t)y_j^2(t) > \quad = \quad 0, \quad \text{for } i > j. \tag{12}$$

It can be shown that (11) and (12) implies that

$$\begin{aligned} < \mathbf{y}(t)\mathbf{y}^T(t) > &= \Lambda_1, \\ < \mathbf{y}(t)\mathbf{f}^T(\mathbf{y}(t)) > &= \Lambda_2 \end{aligned} \tag{13}$$

where Λ_2 is a nonsingular diagonal matrix. By Theorem 1, the source signals can be recovered when (13) is satisfied.

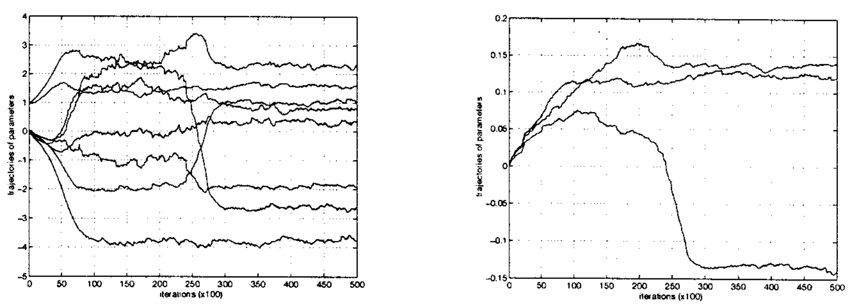
4. Computer Simulation

The computer simulation is conducted to evaluate the performance of the learning algorithm (9), (10). The overall system, $H = (I + W)^{-1}PA$, is supposed to be generalized permutation matrix at desirable equilibria.

Three different sources are drawn from the same one-sided exponential distribution with unit variance and zero mean. The mixing matrix A is chosen randomly as

$$A = \begin{bmatrix} 0.2909 & 0.5046 & 0.5968 \\ 0.0484 & 0.3671 & 0.8085 \\ 0.0395 & 0.9235 & 0.9253 \end{bmatrix}. \tag{14}$$

The learning rates for $P(t)$ and $W(t)$ are .0003 and .00003, respectively. Figure 2 shows the convergence of $P(t)$ and $W(t)$. They are plotted every 100 iterations.



(a) (b)

Figure 2: (a) The convergence of parameters $p_{ij}(t)$; (b) The convergence of parameters $w_{ij}(t)$

The overall transformation matrix H at steady state is

$$H = \begin{bmatrix} -0.0210 & -1.0077 & -0.0041 \\ 0.0123 & 0.0167 & -0.9540 \\ -0.9983 & 0.0192 & -0.0120 \end{bmatrix}. \tag{15}$$

It can be observed that H is very close to $\Pi\Lambda$ (generalized permutation matrix).

5. Conclusion

In this paper, a necessary and sufficient condition for the blind separation of source signals having non-zero skewness, is presented. Then, a neural network is constructed to find a linear transformation which is able to recover the source signals. Both theory and implementation are provided.

Appendix

Proof of Main Theorem:
i) $\Rightarrow$: It is straightforward.
ii) $\Leftarrow$: We can rewrite the equation (5) as

$$(HR_s^{\frac{1}{2}})(HR_s^{\frac{1}{2}})^T = (\Lambda_1^{\frac{1}{2}})(\Lambda_1^{\frac{1}{2}})^T. \tag{16}$$

Then there exits an orthogonal matrix Q such that

$$HR_s^{\frac{1}{2}} = \Lambda_1^{\frac{1}{2}}Q. \tag{17}$$

Hence,

$$H = \Lambda_1^{\frac{1}{2}}QR_s^{-\frac{1}{2}}. \tag{18}$$

Substitute (18) into (6) to obtain

$$QR_s^{-\frac{3}{2}}K_s(Q \circ Q)^T = \Lambda_1^{-\frac{3}{2}}\Lambda_2. \tag{19}$$

Here, we have used the relations, $(DQ)\circ(DQ) = D^2(Q\circ Q)$ and $(QD)\circ(QD) = (Q \circ Q)D^2$ when D is diagonal. Premultiply (19) by Q^T to obtain

$$R_s^{-\frac{3}{2}}K_s(Q \circ Q)^T = Q^T\Lambda_1^{-\frac{3}{2}}\Lambda_2 \tag{20}$$

Since $R_s^{-\frac{3}{2}}K_s$ and $\Lambda_1^{-\frac{3}{2}}\Lambda_2$ are diagonal matrices with non-zero diagonal entries and $(Q \circ Q)^T = Q^T \circ Q^T$, from Lemma 1, $Q = \Pi \overset{\circ}{I}$. Thus, (18) can be written as

$$H = \Lambda_1^{\frac{1}{2}}\Pi \overset{\circ}{I} R_s^{-\frac{1}{2}}. \tag{21}$$

Or,

$$\begin{aligned} H &= \Pi\Pi^T\Lambda_1^{\frac{1}{2}}\Pi \overset{\circ}{I} R_s^{-\frac{1}{2}} \\ &= \Pi\Lambda, \end{aligned} \tag{22}$$

where $\Lambda = \Pi^T\Lambda_1^{\frac{1}{2}}\Pi \overset{\circ}{I} R_s^{-\frac{1}{2}}$ is a diagonal matrix. □

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