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Self Organizing Map for Adaptive Non-stationary Clustering: some experimental results on Color Quantization of image sequences

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Abstract: In this paper we consider the application of the Self Organizing Map to the adaptive computation of cluster representatives (codevectors) over non-stationary data. The paradigm of Non-stationary Clustering is represented by the problem of Color Quantization of image sequences. Experimental results on the Color Quantization of an image sequence show the extreme robustness of the SOM as an adaptive clustering algorithm.

0 Introduction

Cluster analysis and Vector Quantization have applications in signal processing, pattern recognition, machine learning and data analysis [1,2,3,4,5,6]. A vast number of approaches have been proposed to solve these problems, among them Competitive Neural Networks have been proposed as a kind of adaptive partitional methods [7,8,9]. Conventional formulations of Clustering and Vector Quantization assume that the underlying stochastic process is stationary and that a given set of sample vectors properly characterizes this process. Non-stationary processes are dealed with applying a predictive approach to reduce the non-stationary problems to the stationary framework [1]. This paper tries to address the general non-stationary case when there is no acceptable predictive model, and test the robustness of the Self Organizing Map [9] as an adaptive algorithm for the fast search of Non-stationary Clustering solutions. We try to show that the SOM is able for (near) real time application. Therefore, a main restriction that we will impose in their application is a "one pass" adaptation to obtain the representatives at each time step. Other instance of the "one-pass" aproach is [18]. This restriction imposes very strong computational limitations. This work is a continuation of the work reported in [10] where the Simple Competitive Learning and Soft Competition learning rules were applied to the same problem. The present results with the Self Organizing Map clearly improve over the ones reported in [10].

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Color Quantization has been chosen as the application for illustration purposes [11,12]. Color Quantization is easily stated as a Clustering problem and has applications in visualisation [11,12,13], color image segmentation [14], data compression [15] and image retrieval [16]. Although sequences of images (video) lead naturally to the consideration of Non-stationary Clustering problems, the usual approaches consider time invariant distributions of either colors or image blocks, and apply conventional Clustering methods. Some heuristical efforts [15,17] have been reported that try to cope with the time varying characteristics inherent to image sequences. From our point of view, Color Quantization of image sequences summarizes the paradigm of Non-stationary Clustering.

Section 1 gives a working definition for the Non-stationary Clustering problem and the meaning of the adaptive approach in this context. Section 2 comments on the definition of the SOM control parameters used in the experiment. Section 3 discusses the experimental results obtained. Finally, section 4 gives some conclusions and further work

1 Adaptive approach to Non-stationary Clustering/VQ

Conventional formulations of Cluster analysis and Vector Quantization assume that the data is a sample $\aleph = \{\mathbf{x}_1,..,\mathbf{x}_n\}$ of an stationary stochastic process, whose statistical characteristics will not change in time. Non-stationary Clustering and Vector Quantization assume a non-stationary stochastic process that is sampled at diverse time instants. An important remark: we do not assume any knowledge of the time dependencies that could allow a predictive approach [1]. Non-stationary Clustering assumes that the population can be modelled by a discrete time stochastic process $\{\mathbf{X}_t \;\; t = 0,1,..\}$. A working definition of the time varying Clustering problem could read as follows: Given a sequence of samples $\aleph(t) = \{\mathbf{x}_1(t),..,\mathbf{x}_n(t)\}$ of the population obtain a corresponding sequence of partitions of each sample given by a sequence of sets of disjoint clusters $\mathcal{P}(\aleph(t)) = \{\aleph_1(t),..,\aleph_c(t)\}$ that minimizes a criterium function $C = \sum_{t\geq 0} C(t)$ The Non-stationary Vector Quantization design problem can be stated as the search for a sequence of representatives $\mathbf{Y}(t) = \{\mathbf{y}_1(t),..,\mathbf{y}_c(t)\}$ that minimizes an error function (distortion) $E = \sum_{t\geq 0} E(t)$. The squared Euclidean distance is the dissimilarity measure most widely used to define criterium/error functions. The Non-stationary Clustering/VQ problem can be stated as a stochastic minimization problem:

$$\min_{\{\mathbf{Y}(t)\}} \sum_{t\geq 0} \sum_{j=1}^{n} \sum_{i=1}^{c} \left\| \mathbf{x}_j(t) - \mathbf{y}_i(t) \right\|^2 \delta_{ij}(t) \qquad \delta_{ij}(t) = \begin{cases} 1 & i = \underset{k=1,\dots,c}{\operatorname{argmin}} \left\{ \left\| \mathbf{x}_j(t) - \mathbf{y}_k(t) \right\|^2 \right\} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

The proposition of adaptive algorithms to solve this stochastic minimization problem is based in two simplifying assumptions: (1) The minimization of the sequence of time dependent error function can be done independently at each time step. (2) Smooth (bounded) variation of optimal set of representatives at successive time steps. The adaptive application of SOM is done as follows: At time t the initial

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cluster representatives are the ones computed from the sample of the process at time *t-1*. The sample vectors $\aleph(t)=\{\mathbf{x}_1(t),..,\mathbf{x}_n(t)\}$ are presented sequentially as inputs to compute the adaptation equations and to obtain a new set of cluster representatives. A distinctive feature of the experimental work reported below is that we impose a one-pass adaptation at each time step. That means that the sample vectors will be presented only once and that the scheduling of the learning rate and other learning control parameters are adjusted to that time constraint. Color Quantization of video sequences has an obvious time constraint that gives a physical meaning to the search for Clustering algorithms that meet real time constraints.

2 Application of the SOM to Non-stationary Clustering

The general expression of the learning rule for Competitive Neural Networks has the following shape (τ is the order of presentation of the input vectors):

$$\mathbf{y}_i(\tau+1)=\mathbf{y}_i(\tau)+\alpha_i(\tau)\ \vartheta_i(\mathbf{x}(\tau),\mathbf{Y}(\tau))\ [\mathbf{x}(\tau)-\mathbf{y}_i(\tau)] \quad ; \mathbf{x}(\tau)\in\aleph; 1\le i\le c \qquad (2)$$

The Competitive Neural Networks are designed to perform stochastic gradient minimisation of a distortion-like function. In order to guarantee theoretical convergence, the (local) learning rate $\alpha_i(\tau)$ must cope with the conditions:

$$\lim_{\tau\to\infty}\alpha(\tau)=0,\ \sum_{\tau=0}^{\infty}\alpha(\tau)=\infty,\ \text{and}\ \sum_{\tau=0}^{\infty}\alpha^2(\tau)<\infty \qquad (3)$$

However, these conditions imply very lengthy adaptation processes. The sequence of learning rate values proposed below to meet our "one pass" adaptation constraint does not meet the above conditions.

The function $\vartheta_i(\mathbf{x},\mathbf{Y})$ is the so-called *neighbouring function*. According to its definition, the properties of the learning rule equilibria will be different. In this paper we have assumed a 1D topology of the network. The neighbourhoods considered decay exponentially following the expression:

$$\vartheta_i(\mathbf{x}(\tau),\mathbf{Y}(\tau))=\begin{cases}1 & |w-i|\le\left\lceil(\upsilon_0+1)exp\left(\upsilon^{(0)}\tau\, log\left(1/(\upsilon_0+1)\right)/n\right)\right\rceil \\ 0 & otherwise\end{cases} \quad ;1\le i\le c \qquad (4)$$

$$w=\operatorname{argmin}\left\{\|\mathbf{x}(\tau)-\mathbf{y}_k(\tau)\|^2 k=1,..,c\right\}$$

The size of the sample considered at each time instant is *n*. The initial neighbourhood radius is υ_0. The expression ensures that the neighbouring function reduces to the simple competitive case (null neighbourhood) after the presentation of the first $1/\upsilon^{(0)}$ vectors of the sample. In the experiments, the learning rate follows the expression ($\delta_{ij}(k)$ follows the definition given (1)):

$$\alpha_i(\tau)=0.1(1-\tau_i/n)\ \text{where}\ \tau_i=\sum_{k=1}^{\tau}\delta_{ij}(k) \qquad (5)$$

This expression implies that the learning rate decreases proportionally to the number of times that a codevector "wins" the competition. The adaptation induced by the neighbouring function does not alter the local learning rate. The expression (5) also implies that a local learning rate only reaches the zero value if the codevector "wins" for all the sample vectors. Obviously the sequences of the learning rate parameters given by (5) do not comply with the conditions (3) imposed by the convergence of the stochastic gradient approach.

3 Experimental results on the Color Quantization of an image sequence

The sequence of images used for the experiment is a panning of the laboratory taken with an electronic Apple Quicktake camera. Original images have an spatial resolution of 480x640 pixels. Each two consecutive images overlap 50% of the scene. Lack of space impedes the presentation of the distribution of the pixels in the RGB unit cube that will give an straight impression of the non-stationary nature of the data we are dealing with. As reference non adaptive algorithm we have used a variation of the algorithm proposed by Heckbert [11] as implemented in MATLAB. This algorithm has been applied to the entire images in the sequence in two ways. Figure 1 shows the distortion results of the Color Quantization of the experimental sequence to 16 colors based on both applications of the Heckbert algorithm. The curve denoted *Time Varying Min Var* is produced assuming the non-stationary nature of the data and applying the algorithm to each image independently. The curve denoted *Time Invariant Min Var* comes from the assumption of stationarity of the data: the color representatives obtained for the first image are used for the Color Quantization of the remaining images in the sequence. The gap between those curves gives another indication of the non stationarity of the data. Also this gap defines the response space left for truly adaptive algorithms.

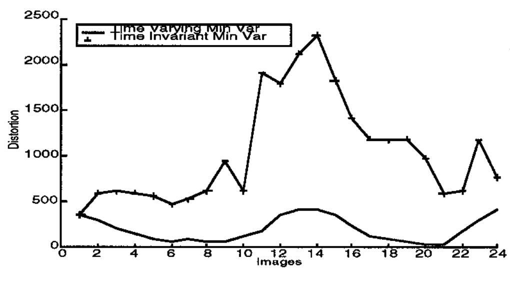

Figure 1. Reference distortion values obtained with the application of the *Time Varying MinVar* and *Time Invariant MinVar* algorithm for 16 colors

Figure 2 shows the results of the application of the Self Organizing Map under several definitions of the sample size and neighbouring function. Random pixel samples of sizes 25600, 6400 and 1600 were used to compute figs 2a,b, figs 2c,d and figs 2e,f, respectively. Regarding the neighbouring function, Figs 2a,c,e have neighbour parameters $\upsilon_0 = 1$ and $\upsilon^{(0)} = 2$; figs 2b,d,f have neighbour parameters $\upsilon_0 = 8$ and $\upsilon^{(0)} = 4$. Each figure shows the distortion results of the Color Quantization of the entire sequence obtained applying the SOM starting from various initial color representatives: the Heckbert color representatives for image #1 (Matlab), a threshold based selection of the sample of image #1 (Umbral), random points in the RGB cube (Random 1) and a random selection of the sample of image #1 (Random 2).

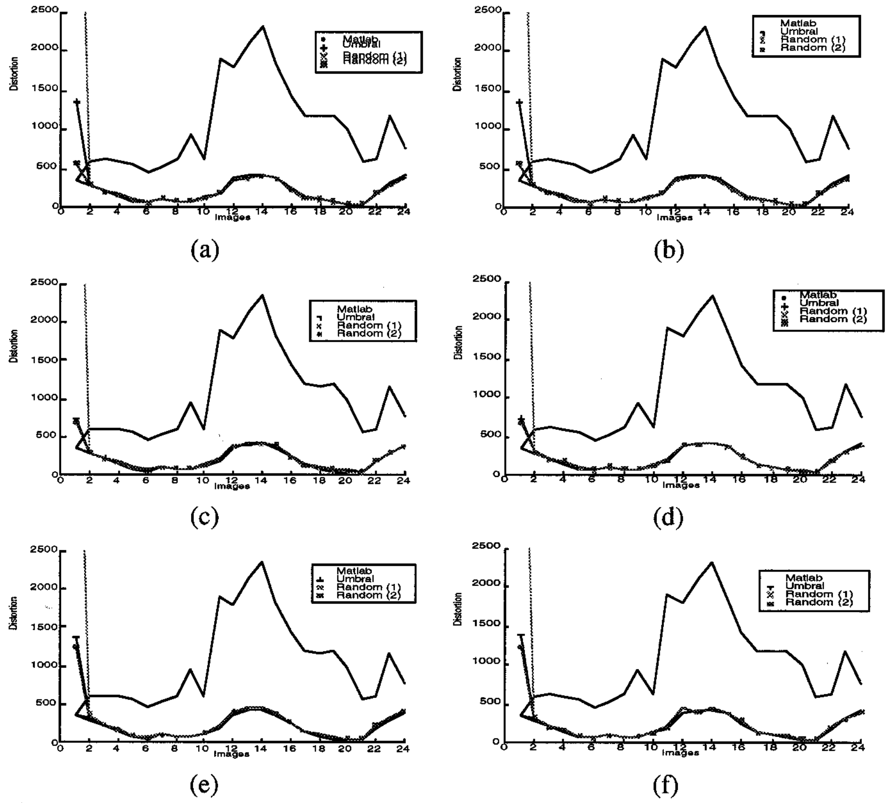

Figure 2. Distortion results of the Color Quantization of the experimental sequence with the color representatives computed adaptively by the SOM for various initial condition, sample sizes and neighbour parameters

The inspection of the figures shows that the SOM obtains optimal results in all the cases and that it is insensitive to the initial conditions. In all cases, after the adaptation over the sample of image #1, the SOM converges to the optimal representatives. It is well known that the SOM operation involves two phases: topological reorganisation and fine tuning. The Matlab initial color representatives are well ordered so that the SOM does not need to perform a reorganisation of the network topology. For the other bad initial conditions, the initial color representatives are disordered, so the adaptation process for image #2 involves also the topological reorganisation of the network. Once this reorganisation is accomplished for image #2, the correct topological order for successive images is the same, so that no new reorganisation occurs. The results obtained demonstrate that SOM is able to perform a very fast topological reorganisation of the color representatives. This adds to the astonishing robustness of the SOM as an adaptive clustering algorithm.

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4 Conclusions and further work

This work tries to emphasize the inherent adaptive nature of the Self Organizing Map to time varying conditions, in particular as a handy tool for the adaptive computation of cluster representatives in the setting of Non-stationary Clustering. The paradigm of Non-stationary Clustering is summarized in the case of Color Quantization of image sequences. The experimental results show the astonishing robustness of the SOM, that gives very good results for all the initial conditions tested, under various settings of the sample size and neighbour parameters. Further experiments are on the way. We strongly believe that the general topic of Non-stationary Clustering is a an open research track with many potential applications, of which the Color Quantization of image sequences is an excellent example.

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