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Anticipated synchronization in neuron models

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Abstract. We study the regime of anticipated synchronization in unidirectionally coupled model neurons subject to a common external aperiodic forcing that makes their behavior unpredictable. We show numerically and by implementation in analog hardware electronic circuits that, under appropriate coupling conditions, the pulses fired by the slave neuron anticipate (i.e. predict) the pulses fired by the master neuron. This anticipated synchronization occurs even when the common external forcing is a white noise.

1 Introduction

Synchronization of nonlinear systems is a fascinating subject that has been extensively studied on a large variety of physical and biological systems[1]. While synchronization of oscillators goes back to the work by Huygens, the last decade has witnessed an increased interest in the topic of synchronization of chaotic systems [2].

Recently, Voss [3] has discovered a new scheme of synchronization, called "anticipated synchronization". Voss has shown that by using appropriate delay lines it is possible to synchronize two unidirectionally coupled systems in such a way that the *slave* system, $\mathbf{y}(t)$, predicts the behavior of the *master* system, $\mathbf{x}(t)$. One of the coupling scheme was considered:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t)) \\ \dot{\mathbf{y}}(t) &= \mathbf{f}(\mathbf{y}(t)) + \mathbf{K}[\mathbf{x}(t) - \mathbf{y}(t-\tau)]. \end{aligned} \tag{1}$$

$\mathbf{f}(\mathbf{x})$ is a function which defines the *autonomous* dynamical system under consideration, $\mathbf{K}$ is the coupling strength and τ is a delay time. It is easy to see that the manifold $\mathbf{y}(t) = \mathbf{x}(t+\tau)$ is a solution of the equations, what becomes more remarkable when the dynamics of the master system $\mathbf{x}$ is "intrinsically

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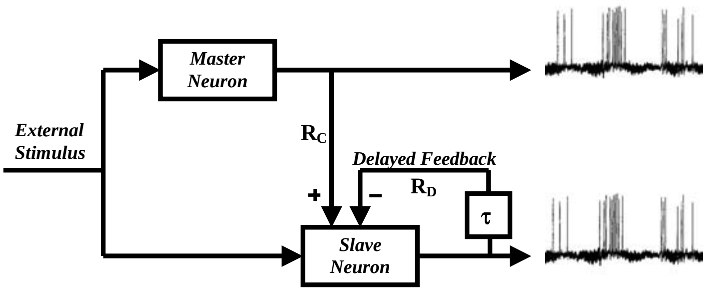

Figure 1: Schematic diagram of two model neurons coupled in a unidirectional configuration, subjected to the same external forcing and with a feedback loop (with a delay time τ) in the slave neuron.

unpredictable", as it is the case of a chaotic system. We study numerically and experimentally the regime of anticipated synchronization in excitable *non-autonomous* systems. In our case the intrinsic unpredictability of the behavior of the dynamical system $\mathbf{x}$ does not arise from a chaotic dynamics, but from the existence of an external random forcing. We consider the coupled systems

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t)) + \mathbf{I}(t) \\ \dot{\mathbf{y}}(t) &= \mathbf{f}(\mathbf{y}(t)) + \mathbf{I}(t) + \mathbf{K}[\mathbf{x}(t) - \mathbf{y}(t-\tau)], \end{aligned} \tag{2}$$

where $\mathbf{I}(t)$ represents a common aperiodic external forcing. Notice that $\mathbf{y}(t) = \mathbf{x}(t+\tau)$ is no longer an exact solution of the equations. We show that under appropriate coupling conditions there can be a very good correlation between $\mathbf{y}(t)$ and $\mathbf{x}(t+\tau)$ which, in practice, allows the prediction of the future behavior of $\mathbf{x}(t)$ with a high degree of accuracy.

Specifically, we have considered models of sensory neurons. Sensory neurons transform external stimuli signals as pressure, temperature, electric pulses, etc., into trains of action potentials, usually referred to as 'spikes' or 'firings'. Their behavior is typical of excitable systems: if the forcing is above a certain threshold, the neuron fires a pulse, and after the firing, the recovery process produces an absolute refractory time during which a second firing cannot occur. In general, sensory neurons work in a noisy environment. As a consequence, the time intervals between spikes contain a significant random component, and random spikes often occur even in the absence of stimuli. The topics of synchronous oscillations and noise have received much attention (see, e.g., [4]), since it has been suggested that synchronous firing activity of sensory neurons might be a part of higher brain functions and a method for integrating distributed information into a global picture [5]. Here we find that the interplay of coupling, delayed feedback, and common noise can lead to anticipated synchronization. We illustrate this effect in the well known FitzHugh-Nagumo and Hodgkin-Huxley neuron models. By coupling two of such systems in an unidirectional configuration as represented in the system (2), we find that when

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both subsystems are subjected to the same external random forcing, the slave fires the same train of spikes as the master, but at a certain amount of time earlier.

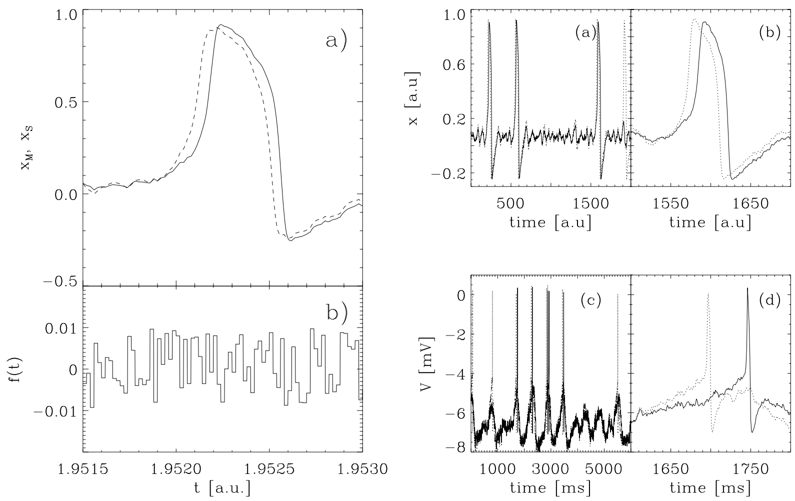

Figure 2: Left part of the panel shows the pulse obtained from a numerical integration of the FitzHugh-Nagumo set of Eqs.(3-4). The parameters are: $a = 0.139$, $b = 2.54$, $\epsilon = 0.008$, $K = 0.15$. The external forcing $I(t)$ (displayed in figure (b)) is a random amplitude noise of characteristic time $T = 2$, mean value $I_0 = 0.03$ and amplitude $D = 0.01$. Notice (figure (a)) that the pulse of the slave system $y_1(t)$ (dashed line) anticipates the pulse of the master system $x_1(t)$ (solid line) by a time approximately equal to the time delay $\tau = 4$. The right part of the panel shows trains of spikes obtained from numerical simulations of models of unidirectionally coupled neurons subjected to the same Gaussian white noise with mean I_0 and correlations $\langle [I(t) - I_0][I(t') - I_0] \rangle = 2D\delta(t - t')$:(a) and (b) show time serie and one firing out of the time serie of FitzHugh-Nagumo neurons, Eqs. (3-4). The parameters are $a = 0.139$, $b = 2.54$, $\epsilon = 0.008$, $I_0 = 0.03$, $K = 0.03$, $\tau = 10$, $D = 2.45 \times 10^{-5}$, (c) and (d) show time serie and one firing out of the time serie of Hodgkin-Huxley neurons, Eqs. (5) with $K = 0.03$ ms^{-1}, τ=50 ms, and D=0.5 mV^2/ms; all other parameters as in [6] ($T = 6$ C, $V_l = -75$ mA in the notation of that paper).

2 Numerical results

First we show results based on the two-dimensional FitzHugh-Nagumo model with variables $\mathbf{x} = (x_1, x_2)$. The fast variable, x_1, is associated with the acti-

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vator, and the slow recovery variable, x_2, is associated with the inhibitor. The equations for the master $\mathbf{x} = (x_1, x_2)$ and the slave $\mathbf{y} = (y_1, y_2)$ systems, under unidirectional coupling are, respectively (see the schematic diagram shown in Fig. 1):

$$\begin{aligned} \dot{x_1} &= -x_1(x_1 - a)(x_1 - 1) - x_2 + I(t) \\ \dot{x_2} &= \epsilon(x_1 - bx_2) \end{aligned} \tag{3}$$

and

$$\begin{aligned} \dot{y_1} &= -y_1(y_1 - a)(y_1 - 1) - y_2 + I(t) + \\ &\quad +K[x_1(t) - y_1(t - \tau)] \\ \dot{y_2} &= \epsilon(y_1 - by_2) \end{aligned} \tag{4}$$

where a, b, and ϵ are constants, K is the coupling strength, τ is a delay time (associated to an inhibitory feedback loop in the slave neuron) and $I(t)$ is an one-dimensional external forcing added to the fast variables of two systems $\mathbf{x}$ and $\mathbf{y}$. Note that only the fast variables of the two systems are coupled. If the external forcing is constant and above threshold, for appropriate values of K and τ the master system fires pulses periodically and the coupling induces a constant time shift τ between master and slave spikes. We did not consider a constant value of $I(t)$ but on the contrary we have considered different types of random external forcing. The first one corresponds to a random process whose amplitude switches after each time period T to a new random value chosen uniformly in $[I_0 - D, I_0 + D]$, where D is the noise intensity. We chose I_0 very close to (but below) the firing threshold of the excitable system. It would appear at first thought that with this type of external forcing the behavior of the master system can be easily predictable. However, there are two main factors that make the system response unpredictable: if the effect of the perturbation is not strong enough the system does not fire a pulse; besides, the system has a refractory time during which, another firing is not possible. Figure 2(left (a-b)) shows that anticipation occurs. After an initial transient time the two systems synchronize such that the slave system anticipates the firings of the master system by a time interval τ.

The same qualitative results are found with other types of external forcing such as colored or even white noise. Figures 2(right (a-b)) display the spikes of the master and slave systems when $I(t)$ is Gaussian white noise. Sometimes the slave system makes an error in anticipating the master firings i.e. it might fire an "extra" pulse, which has no corresponding pulse in the train of pulses fired by the master. Notice that in Fig. 2(right (a)) an error at about $t = 1900$ occurs. Not surprisingly, we find that the longer the anticipation time τ, the larger the number of errors. However, for a given anticipation time, the number of errors can be reduced considerably if a "cascade" of an adequate number of slave neurons is considered. Next we show simulations based on a more realistic model, namely the model of electro-receptors proposed by Braun et. al [6]. This model is a modification of the Hodgkin-Huxley neuron model: $C_M \dot{x} = -i_{Na} - i_K - i_{sd} - i_{sr} - i_l$, where x is the potential voltage across the membrane and C_M is the capacitance; i_{Na} and i_K are the fast sodium and potassium currents, i_{sd} and i_{sr} are additional slow currents, i_l is a passive leak

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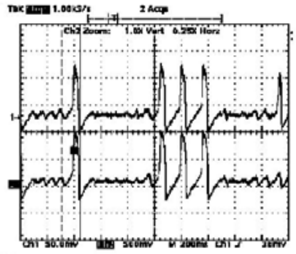

Figure 3: Experimental train of spikes that shows anticipation in the spikes fired by the slave neuron. The anticipation time is approximately 14 ms.

current. For details and functional dependence of the currents on the voltage x and other factors (as temperature) see [6].

We extend the model to account for two unidirectionally coupled neurons, with a delayed feedback loop in the slave neuron, and subject to a common external forcing $I(t)$, in the same way as in the FitzHugh-Nagumo model, e.g., the equations for the master, x, and for the slave, y, neurons are:

$$\begin{aligned} C_M\dot{x} &= -i^x_{Na} - i^x_K - i^x_{sd} - i^x_{sr} - i^x_l + I(t) \\ C_M\dot{y} &= -i^y_{Na} - i^y_K - i^y_{sd} - i^y_{sr} - i^y_l + I(t) \\ &+ K[x(t) - y(t-\tau)] \quad . \end{aligned} \tag{5}$$

Figures 2(c-d) display the results when the common external forcing $I(t)$ is a Gaussian white noise. We chose parameters such that in the absence of forcing there are no spikes (subthreshold, noise-activated firing regime). The behavior observed is qualitatively the same as in the FitzHugh-Nagumo model (the slave neuron anticipates the fires of the master neuron), which indicates that the anticipation phenomenon is general and model independent. Remarkably, in this model the anticipation time can be larger than the pulse duration. To assess the robustness of the anticipated synchronization observed in the numerical simulations, we have implemented the FitzHugh-Nagumo model in analog hardware and constructed two coupled electronic neurons. The detailed description of the electronic implementation can be found in [7]. Similar electronic neurons have been implemented in [8], where it was shown that their behavior is very similar to that of biological neurons: when interfaced to biological neurons, hybrid circuits, with the electronic neurons taking the place of missing or damaged biological neurons, could function normally. Our electronic coupled neurons behave very similar as in the numerical simulations. For an appropriate value of the coupling, we observe that, after a transient, the master and slave electronic neurons synchronize in such a way that the slave neuron anticipates the fires of the master neuron by a time interval approximately equal to the delay time τ of the feedback mechanism. Figure 3 shows a typical spike train. Without coupling and feedback the neurons fire pulses which are, in general, desynchronized (due to the small mismatch between the circuits).

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3 Conclusions

To summarize, we have studied the regime of anticipated synchronization in coupled systems exhibiting neuronal-type excitable behavior, when they are driven by common external aperiodic forcing. We have shown that under appropriate conditions, the slave system can anticipate the random spikes of the master system. This is despite of the fact that the anticipated synchronization manifold is not a solution of the equations. We have simulated numerically the coupled neurons with the FitzHugh-Nagumo and a modified Hodgkin-Huxley models and we have considered different types of random forcing. The FitzHugh-Nagumo model was also implemented in analog hardware, showing that the anticipation phenomenon is very general and robust. Our results show that in coupled model neurons with coexistance of noise and delayed feedbacks, a new interesting and unexpected phenomena might appear. We hope that our findings will stimulate to search for anticipated synchronization in biological systems and to invastigate the associated effects which such synchronization might produce.

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