

# A Neural Network that helps building a Nonlinear Dynamical model of a Power Amplifier

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**Abstract.** This paper presents a new neural network-based model that can be applied to characterize the nonlinear dynamical behavior of power amplifiers. We use a time-delayed feed-forward neural network to make an input-output time-domain characterization, that can provide also an analytical expression (as a Volterra Series model) to predict the amplifier response to multiple power levels. Simulation results that validate our proposal are presented.

## 1 Introduction

The nonlinear analysis of electronic systems often requires an analytical model for each nonlinear element (i.e. an equation representing the input-output relationship), that allows to draw conclusions about the system performance. This approach aims to extract a nonlinear relationship in order to build an input-output model able to generalize the nonlinear dynamic behavior of an electronic component, for input waveforms not used in the characterization set. This procedure is based on the known physical behavior of the modeled device that dictates the equivalent circuits model topology [1]. The process of converting measured data into equations relies on curve-fitting techniques [2]. However, many of the most common techniques are useful where data trace is well behaved over a defined independent variable range and where behavior of an object is known to follow a specific mathematical model, but problems arise when the object's internal parameters make that the data trace exhibit sharp inflections. In that case, common curve-fitting techniques become useless and appears a clear need for a new curve fitting procedure that provides smoothness and continuity through plotted trace having sharp inflections. We claim that a technique that could overcome this problem could be the use of neural networks. In fact, the neural approach for electronic device modeling has received increasing attention, especially in recent years [3][4], since their training procedure needs only simulation or measurements data of the physical circuit under study. A disadvantage of the neural network approach is that it does not provide an analytical expression, often needed when simulating electronic devices. If the time domain approach is chosen in order to characterize the memory effects adding enough time-delayed inputs to the input-output relation, the question is how to learn the nonlinear behavior response to different input power levels, i.e. in the case of a power amplifier. The answer is that time-delayed neural networks can learn a dynamic nonlinear behavior [5][6], if they

are trained with input-output time-domain data samples at different power levels, simultaneously.

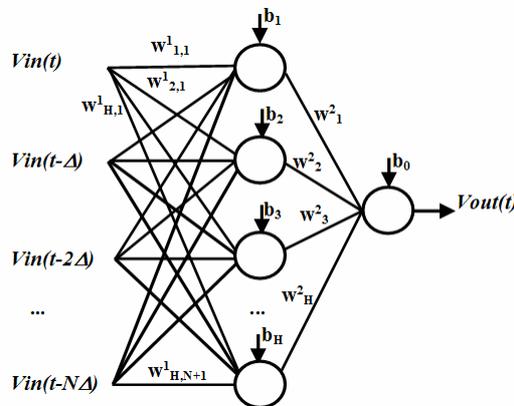
In this paper we show a new application of the procedure presented in [6] to the building of two dynamic nonlinear models of a power amplifier, both a neural network-based model and an analytical Volterra Series model. We present here a new neural network model (based on cubic activation functions) that helps building a Volterra Series for an electronic device, in particular, a power amplifier. The organization of the paper is the following: in the next Section, the neural network-based model of the device is shown. In Section 3 the building of a Volterra Series from the Neural Network parameters is presented. In Section 4 validation results from simulations can be found. Finally, the conclusions appear in Section 5.

## 2 Neural Network model

The neural network used to model the amplifier is a feed-forward time-delayed neural network with three layers, the input time-domain voltage samples and their delayed replies, a nonlinear hidden layer and a linear output. The architecture is shown in Fig. 1, and Eq. 1 and 2 are the corresponding input-output analytical expressions, depending on the type of activation function chosen for the hidden neurons: hyperbolic tangent function or a cubic polynomial, respectively.

$$Vout(t) = b_0 + \sum_{h=1}^H w_h^2 \tanh\left(b_h + \sum_{n=0}^N w_{h,n}^1 Vin(t - n\Delta)\right) \quad (1)$$

$$Vout(t) = b_0 + \sum_{h=1}^H w_h^2 \left(b_h + \sum_{n=0}^N w_{h,n}^1 Vin(t - n\Delta)\right)^3 \quad (2)$$



**Fig. 1.** Time-delayed feed-forward Neural Network model for a power amplifier

In the case of modeling a nonlinear power amplifier, the inputs to the device would be sinusoidal voltage waveforms, that would be amplified at the output according to the amplifier gain. The input and output waveforms are expressed in terms of their samples in the time domain. The memory depth (N) and has been chosen in order to adequately represent the bandwidth of the model. The number of hidden neurons (H) is chosen to perform the best fitting to input-output data without over-fitting. The neural network is trained with a back-propagation algorithm, for one hundred epochs, based on the Levenberg-Marquardt algorithm for network

parameters optimization. The input-output training data sets have been built with cascading samples obtained from each source power level. The neural network has been trained with all the data, simultaneously. This allows to obtain a nonlinear model able to characterize, at the same time, the low and high nonlinear distortion, for all the input power levels of interest. As well as the time-delayed feed-forward neural network-based model can be used to represent a nonlinear device, if trained with the adequate data, also an analytical expression for the model can be built, as a Volterra series expansion, calculated in function of the Neural Network model parameters. This is explained in the next Section

### 3 Non-linear Dynamical model: the Volterra Series

A non-linear dynamical system can be represented exactly by a converging infinite series (Eq.3), that reports the dynamic expansion of a single-input single-output system [7]. This equation is known as the Volterra series expansion and it is extensively used in electronics to model nonlinear dynamical behavior in electronic devices [8]. The coefficients  $h_0, h_1, h_2, \dots, h_n$  are known as the Volterra kernels of the system. In general,  $h_n$  is the  $n^{\text{th}}$  order kernel of the series [9].

$$y(t) = h_0 + \sum_{k=0}^{\infty} h_1(k)x(t-k) + \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} h_2(k_1, k_2)x(t-k_1)x(t-k_2) + \dots \quad (3)$$

In the case of a Volterra expansion for a power amplifier, the input  $x(t)$  would be the input voltage  $V_{in}$  which is applied to the device and the output  $y(t)$  would be the output voltage  $V_{out}$  that the device amplifies. This amplification is generally done for a range of power levels, not only one point, according to the device amplification gain. The Volterra kernels allow the inference of device characteristics of great concern for the microwave designer. However, the number of terms in the kernels of the series increases exponentially with the order of the kernel. Moreover, at microwave frequencies, suitable instrumentation for the measurement of the kernels is still lacking [10]. In spite of this drawback, the Volterra series is used for microwave circuit design, by means of complex and time-consuming analytical or numerical calculations [11]. There have been several proposals for kernels calculation with different, often non standard, neural networks topologies [12][13]. However, all of these approaches propose the use of non standard neural models and learning algorithms, which makes difficult their use, and they do not offer a simple analytical solution derived from the network which could be implemented inside a circuits simulator.

We have found a method in [6] for generating the Volterra series that models a nonlinear FET transistor, using the weights and bias values of a neural network. The procedure, which was applied to a function depending on two or even more input variables, is based on a time-delayed feed-forward neural network having hyperbolic tangent activation functions in the hidden layer (Eq. (1)). This output has to be developed as a Taylor series around the bias values of the hidden nodes. The hyperbolic tangent function derivatives have to be calculated and after accommodating common terms, the Volterra kernels can be easily recognized, kernels that can be combined to form a Volterra Series Model representation of the device under study. However, in this paper we propose a further simplified way of calculating the kernels, avoiding the need for applying a Taylor expansion and the calculation of derivatives. Our proposal is to use cubic activation functions in the

hidden neurons as shows Eq. 2. In this case, the procedure to follow is simply to develop and to distribute the polynomials. After common factoring, the kernels are immediately identifiable, as shows Eq. 4.

$$\begin{aligned}
 Vout(t) = & b_0 + \sum_{h=1}^H w_h^2 (b_h)^3 + \left[ \sum_{h=1}^H w_h^2 w_{h,1}^1 3(b_h)^2 \right] Vin(t) + \left[ \sum_{h=1}^H w_h^2 w_{h,2}^1 3(b_h)^2 \right] Vin(t - \Delta) + \dots + \\
 & \left[ \sum_{h=1}^H \sum_{i=1}^H w_h^2 w_{i,1}^1 w_{i,1}^1 3b_h \right] Vin(t)^2 + \left[ \sum_{h=1}^H \sum_{i=1}^H w_h^2 w_{i,2}^1 w_{i,2}^1 3b_h \right] Vin(t - \Delta)^2 + \dots + \\
 & \left[ \sum_{h=1}^H w_h^2 w_{h,1}^1 w_{h,2}^1 6b_h \right] Vin(t) Vin(t - \Delta) + \dots + \\
 & \left[ \sum_{h=1}^H \sum_{i=1}^H w_h^2 w_{i,1}^1 w_{i,1}^1 w_{i,1}^1 \right] Vin(t)^3 + \left[ \sum_{h=1}^H \sum_{i=1}^H w_h^2 w_{i,2}^1 w_{i,2}^1 w_{i,2}^1 \right] Vin(t - \Delta)^3 + \dots
 \end{aligned} \quad (4)$$

Looking at the terms between brackets in (4), the Volterra Kernels of a Volterra Series expansion can be recognized. It corresponds in this case to the Volterra model of the nonlinear dynamic behavior in a power amplifier. We present here in Eq. 5, 6, 7 and 8 new formulas to calculate up to the 3<sup>rd</sup> order Volterra kernels, using the connection weights and hidden neurons bias values of a neural network model like the one presented in Fig. 1, having cubic activation functions in the hidden layer. In the next Section the validation of the model is presented.

$$h_0 = b_0 + \sum_{h=1}^H w_h^2 (b_h)^3 \quad (5)$$

$$h_1(k) = \sum_{h=1}^H w_h^2 w_{h,k}^1 3(b_h)^2 \quad (6)$$

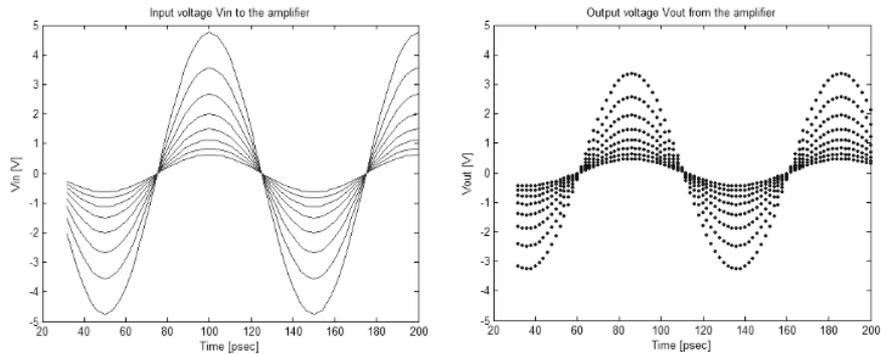
$$h_2(k_1, k_2) = \begin{cases} \sum_{h=1}^H w_h^2 w_{h,k_1}^1 w_{h,k_2}^1 6b_h \\ \sum_{h=1}^H w_h^2 w_{h,k_1}^1 w_{h,k_2}^1 3b_h, (k_1 = k_2) \end{cases} \quad (7)$$

$$h_3(k_1, k_2, k_3) = \begin{cases} \sum_{h=1}^H w_h^2 w_{h,k_1}^1 w_{h,k_2}^1 w_{h,k_3}^1 3b_h \\ \sum_{h=1}^H w_h^2 w_{h,k_1}^1 w_{h,k_2}^1 w_{h,k_3}^1 b_h, (k_1 = k_2 = k_3) \end{cases} \quad (8)$$

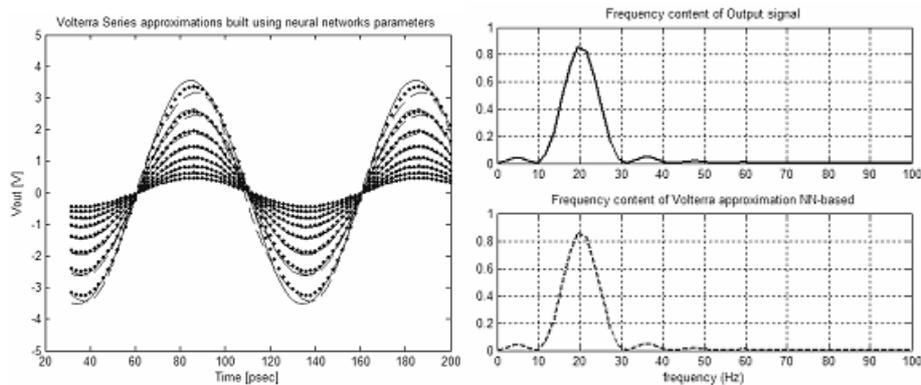
#### 4 Model training and validation

The input/output data used to train the neural network model are shown on Fig. 2. Eight different power levels are used altogether to train the feed-forward network, The starting input power is 0 dBm, the step is 2.5 dBm and the stop power is 20 dBm, yielding eight input powers tested with the device. The inputs to the model are the voltage signal  $Vin$  at the present instant, and four previous delayed samples. Several networks configuration were tried, changing the number of hidden neurons, training the networks models for 100 epochs. After that, the network parameters (weights and bias values) have been used as was explained in Section 3, to build the Volterra Series

model for the amplifier, including up to the 3<sup>rd</sup> order kernels in the approximations. The results are shown in Fig. 3 and 4.



**Fig. 2** Input (solid line) and Output (dotted line) sinusoidal waveforms for a power amplifier. Each line corresponds to a different power level



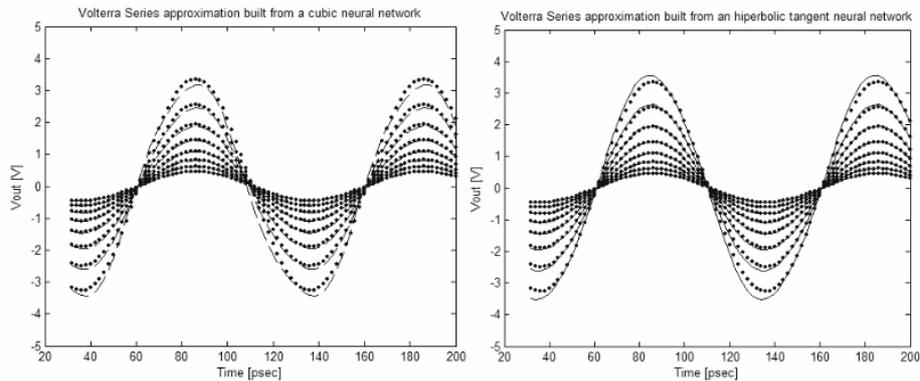
**Fig. 3** Left figure: simulation results after training the neural networks with the original data (dotted line) and building afterwards the Volterra series model up to the 3<sup>rd</sup> order approximation, from a cubic network (double solid line) and from an hyperbolic tangent network (solid line). Right figure: Fourier transform of the output signal at input power level 0 dBm, vs. the Volterra series model obtained from both cubic and hyperbolic tangent approximations.

In any case, the Volterra models extracted in this way are almost totally equivalent among them (you can hardly distinguish one from another, which happens not only in the time domain but also in the frequency domain as shows Fig. 3) and perform almost the same degree of accuracy as the neural network model itself. The best approximation was found with a neural network with 10 hidden neurons, where the mean square error (mse) error in the hyperbolic tangent Volterra approximation is  $5e-03$  and the mse error in the cubic Volterra approximation is  $8e-03$ . As can be noticed, both approximations are very close, and model quite well the original behavior.

## 5 Conclusions

In this paper we have presented a new method for the building of the Volterra series of a nonlinear dynamical power amplifier, using a neural network model trained with

data from eight different power levels simultaneously. Even though the new method, that proposes the use of cubic activation functions in the hidden layer to further simplify the procedure, has little more error than the original proposal with hyperbolic functions, it is faster and simpler to calculate because no derivatives have to be derived, simply the cubic polynomials have to be developed.



**Fig. 4** The first plot shows the Volterra series approximation built from a cubic neural model(double solid line) and the second plot shows a Volterra series approximation built from an hyperbolic tangent neural model(solid line).

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