ESANN'2006 proceedings - European Symposium on Artificial Neural Networks
Bruges (Belgium), 26-28 April 2006, d-side publi., ISBN 2-930307-06-4.

A Simple Idea to Separate Convolutive Mixtures in an Undetermined Scenario

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Abstract. We consider a blind separation problem for undetermined mixtures of two BPSK signals in a multi-path fading channel. We use independence and frequency diversity of the two source signals to identify mixture parameters, estimate Pulse Shaping Filters (PSF) and channel responses, as well as to extract both binary sequences from only one observation. Presented method uses gradient descent algorithm to directly adopt the symbols, which are then used as feedback sequence for PSF roll-off factor identification as well as for channel equalization.

1 Introduction

The problem of Blind Source Separation (BSS) has been intensively studied in the literature and many effective solutions have been proposed in the case of instantaneous mixtures (memoryless channel) [1–4] and convolutive mixtures (channel effects can be considered as a linear filter) [5–9]. Most of the proposed algorithms deal with an undercomplete case (the number of sensors is equal or greater to the number of sources). For more sources than mixtures [10–15], the BSS problem is said to be overcomplete (undetermined) and is ill-posed.

In general, separation of overcomplete mixtures remains still a great challenge for the scientific community. Even though the methods of identifying instantaneous mixing coefficients for n sources have been developed [11], they need at least 2 sensors. The same assumptions limit the method of separating undetermined mixtures proposed in [10] (two or more sensors).

In our contribution, we present a new idea to deal with the case of one sensor and two sources (undetermined problem). The mixture is considered to be convolutive (multipath fading channel) and the sources to be linearly modulated digital signals. Such a scenario can be found in a cellular phone reception, satellite transmissions, as well as in military communications (eg. signal interception, jamming or counter-measure). We propose methods of identifying mixture parameters, estimating PSFs and channel responses, as well as extracting both source signals. We consider realistic scenario of very small (compared to signals' bandwidths) shift between carrier frequencies (eg. shift: 20 Hz, bandwidths: 1600 Hz, sampling frequency: 8000 Hz), a class of raised-cosine PSF, and real-life fading channels of type Rayleigh and Rice.

2 Signal Model

Let us consider a linear, convolutive mixture $x(t)$ of two BPSK-type signals

$$x(t) = s_1(t)\mathrm{e}^{i(\omega_1 t+\varphi_1)} + s_2(t)\mathrm{e}^{i(\omega_2 t+\varphi_2)} \tag{1}$$

where ω_k and φ_k are carrier frequencies and equivalent phases (sum of carrier and mixing coefficient phases), and the source signals $s_k(t)$ are defined as

$$s_k(t) = a_k c_k(t) * \sum_l d_{kl} p_k(t - lT_k) \tag{2}$$

where a_k are unknown real mixing coefficients, $c_k(t)$ are real channel impulse responses, $d_{kl} \in \{+1, -1\}$ (BPSK case) are equiprobable, independent and identically distributed (i.i.d.) random sequences, $p_k(t)$ are PSFs belonging to a class of raised-cosine filters, and T_k are symbol durations.

We assume that source signals d_{kl} are independent of each other and carrier frequencies ω_k are distinct and can be estimated [16] or (with some modifications) [17]. We consider scenario with small (compared to Baud Rates) frequency shifts (as in intercepting military transmissions in "double talk mode") that any separation method based on signal filtering [18,19] can't be applied. Finally, we assume that timing parameters T_k are already estimated [20,21].

3 Estimation of Equivalent Phases

Using frequency diversity along with the independence between source signals, the equivalent phases can be estimated using auxiliary signals defined as

$$Z_k(t) = x(t)\mathrm{e}^{-i\omega_k t} = s_k(t)\mathrm{e}^{i\varphi_k} + s_l(t)\mathrm{e}^{i((\omega_l-\omega_k)t+\varphi_l)}, \quad k,l \in \{1,2\}, k \neq l \tag{3}$$

and the mean values of its squares

$$\begin{aligned}\mathcal{E}\{Z_k^2(t)\} = \mathcal{E}\{s_k^2(t)\}\mathrm{e}^{i2\varphi_k} + \mathcal{E}\{s_l^2(t)\}\mathcal{E}\left\{\mathrm{e}^{i2((\omega_l-\omega_k)t+\varphi_l)}\right\} \\ + 2\mathcal{E}\{s_k(t)s_l(t)\}\mathcal{E}\left\{\mathrm{e}^{i((\omega_l-\omega_k)t+\varphi_k+\varphi_l)}\right\}\end{aligned} \tag{4}$$

where $\mathcal{E}\{x(t)\} = \lim_{T_o\to\infty} \frac{1}{T_o}\int_{-T_o/2}^{+T_o/2} x(t)\,dt$.

Using the independence between the source signals $\mathcal{E}\{s_k(t)s_l(t)\} = 0$, and assuming that differences between the frequencies and/or observation time are big enough $\mathcal{E}\left\{\mathrm{e}^{i2((\omega_l-\omega_k)t+\varphi_l)}\right\} = 0$, one can write above equation as

$$\mathcal{E}\{Z_k^2(t)\} = \mathcal{E}\{s_k^2(t)\}\mathrm{e}^{i2\varphi_k} \tag{5}$$

so the equivalent phases can be estimated by

$$\hat{\varphi}_k = \tfrac{1}{2}\arg\left[\mathcal{E}\left\{Z_k^2(t)\right\}\right] \tag{6}$$

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4 Separation Procedure

4.1 First Approach

Once we estimated carrier frequencies and equivalent phases, one can downconvert received signal $x(t)$ to obtain another auxiliary variables $X_k(t)$

$$X_k(t) = x(t)\mathrm{e}^{-i(\omega_k t+\varphi_k)} = s_k(t) + s_l(t)\mathrm{e}^{i((\omega_l-\omega_k)t+\varphi_l-\varphi_k)} \tag{7}$$

Using the fact that original signals $s_k(t)$ are 1D (real signals) and the observation $x(t)$ spans 2D complex plane, the separation problem can be resolved by decomposing auxiliary signals $X_k(t)$ into its imaginary parts

$$\Im\{X_k(t)\} = s_l(t)\sin((\omega_l-\omega_k)t+\varphi_l-\varphi_k) \tag{8}$$

then, by its simple inversion

$$\hat{s}_1(t) = \frac{\Im\left\{x(t)\mathrm{e}^{-i(\omega_2 t+\varphi_2)}\right\}}{\sin((\omega_1-\omega_2)t+\varphi_1-\varphi_2)} \tag{9}$$

$$\hat{s}_2(t) = \frac{\Im\left\{x(t)\mathrm{e}^{-i(\omega_1 t+\varphi_1)}\right\}}{\sin((\omega_2-\omega_1)t+\varphi_2-\varphi_1)} \tag{10}$$

which exists excepted the particular instants of time

$$t_0(k) = (k\pi+\varphi_1-\varphi_2)/(\omega_2-\omega_1), \quad k = 0,1,2,\ldots \tag{11}$$

4.2 Objective Function

To avoid implementation problems at asymptotic points defined by (11), we propose an objective-function-based approach to separate the sources. To explain this idea, let us suppose that second signal $s_2(t)$ is known. In such case, one can subtract it from the received mixture $x(t)$, and then make a downconversion with a complex exponential $\mathrm{e}^{-i(\omega_1 t+\varphi_1)}$ to obtain a real variable $\varepsilon(t)$

$$\varepsilon(t) = (x(t) - s_2(t)\times\mathrm{e}^{i(\omega_2 t+\varphi_2)})\times\mathrm{e}^{-i(\omega_1 t+\varphi_1)} = s_1(t) \tag{12}$$

In a real-life scenario, we have no idea about the signal $s_2(t)$, and additionally, we have to deal with a noisy case

$$x(t) = c_1(t)*p_1(t)*d_1(t)\mathrm{e}^{i(\omega_1 t+\varphi_1)} + c_2(t)*p_2(t)*d_2(t)\mathrm{e}^{i(\omega_2 t+\varphi_2)} + n(t) \tag{13}$$

where $s_k(t) = c_k(t)*p_k(t)*d_k(t)$ is a simplified notation for signal filtering, and $n(t)$ is complex, Additive White Gaussian Noise (AWGN) with a variance σ^2. In this case, equation (12) becomes

$$\begin{aligned}\varepsilon(t) &= (x(t) - \hat{s}_2(t)\times\mathrm{e}^{i(\omega_2 t+\varphi_2)})\times\mathrm{e}^{-i(\omega_1 t+\varphi_1)} \\ &= s_1(t) + [s_2(t)-\hat{s}_2(t)]\mathrm{e}^{i((\omega_2-\omega_1)t+\varphi_2-\varphi_1)} + n'(t)\end{aligned} \tag{14}$$

where $n'(t)$ is a downconverted version of the original white noise $n(t)$.

By taking imaginary part of this relation, and by computing its variance (block $\mathcal{V}\{\Im\{*\}\}$ in the diagram 1), one can define the following objective function

$$Q(t) = \mathcal{V}\{\Im\{\varepsilon(t)\}\} = [s_2(t) - \hat{s}_2(t)]^2 \sin^2((\omega_2 - \omega_1)t + \varphi_2 - \varphi_1) + \sigma^2 \quad (15)$$

which reaches its minimum $Q_{\min} = \sigma^2$ when $\hat{s}_2(t) = s_2(t)$ for $t \neq t_0$. For values $t = t_0$ (*cf.* equation (11)), the objective function $Q(t)$ doesn't depend on $\hat{s}_2(t)$ and can't be used to find correct estimates.

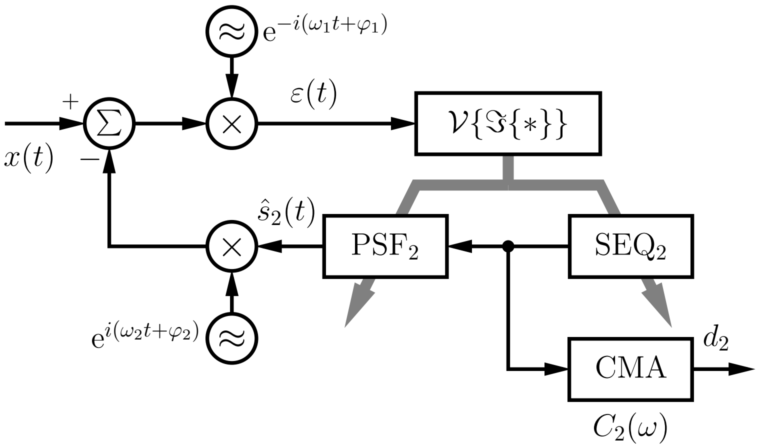

Fig. 1: Scheme of the proposed algorithm.

4.3 Algorithm Implementation

Practical application of the objective function $Q(t)$ to find the estimate of the source signals $\hat{s}_k$ is schematised in the diagram 1.

Firstly, we fix roll-off factor $\alpha = 0.4$ for the raised-cosine filter [22] (block PSF$_2$ in the diagram), and we use the objective function $Q(t)$ to adopt 8 first symbols (block SEQ$_2$ in the diagram). It should be noted that we adopt filtered sequence (binary sequence convolved with a channel: $c_k(t) * d_k(t)$) directly, and we use some kind of redundancy in the data (we adopt 8 real numbers, whereas objective function is calculated using $8 * F_s/F_d$ (F_s – sampling frequency; F_d – Baud Rate) samples. To adopt the sequence we use standard Stochastic Gradient Descent (SGD) algorithm [22, 23], which converge for any $t \neq t_0$ (quadratic function of $\hat{s}_k(t)$). For $t = t_0$, the algorithm doesn't change the initial sequence ($Q(t) = Q_{\min}$ so adaptation doesn't take place).

Next, to speed up the computations, we adopt remaining sequence by adopting 3 symbols at a time (1 current symbol and 2 preceding). After all symbols were estimated (128 according to our contract specifications), we use the objective function once again to adopt the roll-off factor α of the PSF. Finally, we apply Constant Modulus Algorithm (CMA block in the diagram) [24] to equalize the channel ($C_2(\omega)$ in the diagram) and to find estimate of the initial binary sequence (d_2 in the diagram).

To find the second binary sequence, we apply second, identical (excepted the frequencies used to downconvert respective signals) branch (in the diagram only the first branch was visualised) and we make all the computations in parallel. Experimental results have confirmed that using only 8 symbols in the first step, we are able to correctly initialise the algorithm, and that the PSF can be sufficiently identified after all symbols were processed (division of adaptation procedure into two steps: symbol adaptation with guessed PSF, and PSF adaptation with estimated symbols). It should be noted that this algorithm is quite time consuming (≈ 60 s on Pentium 4 HT with 3 GHz clock and 512 MB of RAM) so at the current stage, its real-time implementation is impossible.

5 Conclusions and Perspectives

We consider an undetermined problem of blind separation of two BPSK signals from a convolutive mixture (multi-path fading channel). We use independence and frequency diversity of two source signals to identify mixture parameters, estimate PSFs and channel responses, as well as to extract both binary sequences from only one observation.

It should be noted that the final algorithm is still "under construction", and at the current stage we are unable to give any comparison with existing methods nor show the results of exhaustive simulations (eg. Bit Error Rate (BER) as a function of Signal to Noise Ratio (SNR)). Currently, we are working on extension of the developed algorithm to get around with theoretical limitations (*cf.* equations (9), (10), and (11)), i.e. using estimated PSFs and channel responses to regenerate the original source signals, than apply an objective function (which uses both signals) to readopt original binary sequences. Experimental results, case of general, linear digital modulations ($d_{kl} \in \mathbb{C}$), as well as complex channels (phase shifts taken into considerations) are our current occupation and are the subject of a future publication.

References

[1] S.I. Amari, J.F. Cardoso, *Blind Source Separation – Semiparametric Statistical Approach*, IEEE Transactions on Signal Processing, vol. 45, no. 11, pp. 2692-2700, November 1997.

[2] J.F. Cardoso, P. Comon, *Tensor-Based Independent Component Analysis*, in Signal Processing V, pp. 673-676, 1990.

[3] J.F. Cardoso, B. Laheld, *Equivariant Adaptive Source Separation*, IEEE Transactions on Signal Processing, vol. 44, no. 12, pp. 3017-3030, December 1996.

[4] A. Mansour, N. Ohnishi, C.G. Puntonet, *Blind Multiuser Separation of Instantaneous Mixture Algorithm Based on Geometrical Concepts*, Elsevier Signal Processing, vol. 82, no. 8, pp. 1155-1175, 2002.

[5] N. Delfosse, P. Loubaton, *Adaptive Blind Separation of Convolutive Mixtures*, in ICASSP 96, pp. 2940-2943, Atlanta, USA, May 1996.

[6] A. Mansour, C. Jutten, P. Loubaton, *An Adaptive Subspace Algorithm for Blind Separation of Independent Sources in Convolutive Mixture*, IEEE Transactions on Signal Processing, vol. 48, no. 2, pp. 111-114, February 2000.

[7] A. Mansour, *A Mutually Referenced Blind Multiuser Separation of Convolutive Mixture Algorithm*, Elsevier Signal Processing, vol. 81, no. 11, pp. 2253-2266, November 2001.

[8] P. Comon, L. Rota, *Blind Separation of Independent Sources From Convolutive Mixtures*, IEICE Transactions on Fundamentals of Electronics Communications and Computer Sciences, vol. E86-A, no. 3, March 2003.

[9] J. Antoni, F. Guillet, M. El Badaoui, F. Bonnardot, *Blind Separation of Convolved Cyclostationary Processes*, Elsevier Signal Processing, vol. 85, pp. 51-66, 2005.

[10] P. Comon, O. Grellier, *Non-Linear Inversion of Undetermined Mixtures*, in ICA 99, pp. 461-465, Aussois, France, January 11-15, 1999.

[11] A. Taleb, *An Algorithm for the Blind Identification of N Independent Signals with 2 Sensors*, in ISSPA 01, Kuala-Lampur, Malaysia, August 13-16, 2001.

[12] L. De Lathauwer, B. Moor, J. Vandewalle, *ICA Techniques for More Sources Than Sensors*, IEEE Signal Processing Workshop, Higher-Order Statistics, pp. 116-120, 1999.

[13] T. Lee, M.S. Lewicki, M. Girolami, T.J. Sejnowski, *Blind Source Separation of More Sources Than Mixtures Using Overcomplete Representations*, IEEE Signal Processing Letters, vol. 6, no. 4, pp. 87-90, 1999.

[14] P. Bofill, M. Zibulevsky, *Blind Separation of More Sources Than Mixtures Using Sparsity of Their Short-Time Fourier Transform*, in ICA 00, pp. 87-92, Helsinki, Finland, 2000.

[15] L. Nguyen, A. Belouchrani, K. Abed-Meraim, B. Boashash, *Separating More Sources Than Sensors Using Time-Frequency Distributions*, in ISSPA 01, vol. II, pp. 583-586, 2001.

[16] S.L. Marple, *Digital Spectral Analysis*, Prentice-Hall, 1987.

[17] M. Pedzisz, A. Mansour, *HOS-Based Multi-Component Frequency Estimation*, in EUSIPCO 2005, Antalya, Turkey, 4-8 September 2005.

[18] L. Benaroya, F. Bimbot, *Wiener Based Source Separation with HMM/GMM Using a Single Sensor*, in Report GdR ISIS, Paris, France, 12 June 2003.

[19] J.R. Hopgood, P.J.W. Rayner, *Single Channel Nonstationary Stochastic Signal Separation Using Linear Time Varying Filters*, IEEE Transactions on Signal Processing, vol. 51, no. 11, pp. 1739-1752, July 2003.

[20] G.B. Giannakis, *Linear Cyclic Correlation Approaches for Blind Identification of FIR Channels*, in 28th Asilomar Conference on Signals, Systems and Computers, Pacific Grove, USA, November 1994.

[21] G. Scarano, G. Panci, *On Cyclic Correlation Approaches for Blind Identification of FIR Communication Channels*, in First IEEE Signal Processing Workshop on Signal Processing Advances in Wireless Communications, pp. 81-84, Paris, France, 16-18 April, 1997.

[22] J.G. Proakis, *Digital Communications*, McGraw-Hill, 4th edition, 2001.

[23] S. Haykin, *Adaptive Filter Theory*, Prentice-Hall, 3rd edition, 1996.

[24] C.R. Johnson, P. Schniter, T.J. Endres, J.D. Behm, D.R. Brown, R.A. Casas, *Blind Equalization Using the Constant Modulus Criterion: A Review*, IEEE Transactions on Signal Processing, vol. 86, no. 10, pp. 1927-1950, October 1998.