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Nonlinear dynamics in neural computation

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Abstract. This tutorial reports on the use of nonlinear dynamics in several different models of neural systems. We discuss a number of distinct approaches to neural information processing based on nonlinear dynamics. The models we consider combine controlled chaotic models with phenomenological models of spiking mechanisms as well as using weakly chaotic systems. The recent work of several major researchers in this field is briefly introduced.

1 Introduction

The use of nonlinear dynamics in models of neural systems has been studied for over a decade. Both experimentalists as well as theorists have investigated and proposed different mechanisms which would allow nonlinear dynamics to be used [1, 2]. Although the existence of chaos in neuronal systems appears to be not in doubt [3], the possible role of chaos is still under discussion [4, 5, 6]. In particular, the possible use of chaos at the core of information processing has been considered to be potentially useful [7, 8]. Even though much is now known about chaotic systems, their synchronisation and control, the next step of relating information to a stable state contained in a (controlled) chaotic system appears elusive. (For a detailed explanation of chaotic control and synchronisation see [9, 10, 11, 12, 13]). In this tutorial, we will explore several systems which provide support for the use of controlled chaotic systems as dynamic filters and transient information processing.

2 Emergent behaviour

Some recent developments in chaotic neural models is the application of controlled chaotic systems in autonomous models. The introduction of purely chaotic systems in any neural model is feasible, however, these tend to become either indistinguishable from stochastic systems or have only a particular feature of the chaotic model which is included in the resulting dynamics. Controlling specific unstable periodic orbits, upon presentation of input, such that they are reliably correlated to that particular input seems to be complicated. In many cases targeting the control towards a particular solution requires non-biologically relevant mechanisms.

Instead of applying control of a chaotic system upon input, the control can be employed continuously, in other words, the chaotic system is always under some form of control. The system becomes therefore stable periodic, even though the controlled model is only unstable periodic. The possible advantages are that the system is only semi-stable, i.e. only during the time that control is effective has it stable properties. When the control is not effective, for example when the control function is close to zero, the system does not exhibit chaotic properties but can be perturbed into different trajectories.

2.1 Dynamic patterns

To show how a dynamic behaviour may emerge from controlled chaoticly driven neurons a neuron model been derived from the Hindmarsh-Rose (HR) model [14] but includes a slow recurrent equation which represents the slow calcium exchange between intracellular stores and the cytoplasm [15]. This makes the modified Hindmarsh-Rose model (HR4) more like a chaotic Hodgkin-Huxley (HH) model of stomatogastric ganglion neurons [15]. In addition to the slow calcium current, an additional inactivation current has been added to this model, which competes with the third current to return the system to the equilibrium state. The third equation of the HR4 model is complemented with a fifth equation resulting in the five dimensional Hindmarsh-Rose model (HR5). The effect of the faster inactivation current z_f (4), compared to the slower inactivation current as used in HR4, is that the system tends to burst less. The faster current makes the system return quickly towards the equilibrium where only a larger (re)activation current can cause the system to burst. In this model, the HR5 system allows the temporal separation of spikes by increasing the refactory period. Parameter values are $a = 1$, $b = 3$, $c = 1$, $e = 1$, $f = 5$, $g = 0.0275$, $u = 0.00215$, $s = 4$, $v = 0.001$, $k = 0.9573$, $r = 3.0$, $m = 1$, $n = 1, s_{f_1} = 8$, $s_{f_2} = 1$, $n_f = 4, d_f = 0.5$, with rest-potential $x_0 = 1.605$ and variable input I. With these parameter values the model is stable in the resting potential but shows low dimensional chaos in the bursting patterns.

$$\frac{dx}{dt} = a\,y + b\,x^2 - c\,x^3 - d_s\,z_s - d_f\,z_f + I \tag{1}$$

$$\frac{dy}{dt} = e - f\,x^2 - m\,y - g\,w \tag{2}$$

$$\frac{dz_s}{dt} = u\,(s_s\,(x + x_0) - n_s\,z_s) \tag{3}$$

$$\frac{dz_f}{dt} = u\,((s_{f_1}\,(x + x_0) - s_{f_2}\,x^2) - n_f\,z_f) \tag{4}$$

$$\frac{dw}{dt} = v\,(r(y + l) - k\,w) \tag{5}$$

To introduce controlled chaotic behaviour in either the four dimensional HR4 system or the five dimensional HR5, a scaled and inverted Rössler system has been used [16]. This is necessary because the normal Rössler model has a different time scale from the HR4 model but the scaled variables are proportional to the normal Rössler parameter values. It is possible to map the time scale of the modified Rössler (R3) model to fit the time scale of the HR4 model and use the R3 system to generate patterns. In addition to the scaling, the u_r variable has been inverted to enable the convenient use of this variable as the drive for the HR4 model. Parameter values are $a_r = \frac{1}{75}$, $b_r = \frac{1}{15}$, $c_r = \frac{1}{15}$, $d_r = \frac{1}{50}$, $k_r = -0.57$, $w_r = -\frac{1}{75}$ and $p_r = -1$.

$$\frac{d\,x_r}{d\,t} = -b_r\,y_r - d_r\,u_r \tag{6}$$

$$\frac{d\,y_r}{d\,t} = c_r\,x_r + a_r\,y_r \tag{7}$$

$$\frac{d\,u_r}{d\,t} = p_r\,u_r\,x_r + k_r\,u_r + w_r \tag{8}$$

The R3 system is controlled into an unstable periodic orbit using a chaotic rate control mechanism [17]. This mechanism allows the system to exhibit different periodic orbits by limiting the rate of change of equation (8). The rate control variable σ is only different from 1 if the variables x and u are diverging rapidly, i.e. when the chaotic manifold is stretching or folding. Equation (8) is modified to (10) as shown below. The rate control parameter μ determines the strength of the rate limiting function and the parameter ξ can have different values but is usually $-2 \leq \xi < 0$. This chaotic control mechanism is very effective at stabilising different unstable periodic orbits, but not for any given value of μ and ξ. Typically used values are $\mu = 6$ and $\xi = -1$ or $\xi = -2$.

$$\sigma(x,u) = e^{\dfrac{\xi(xu)}{(u+x+\mu)}} \tag{9}$$

$$\frac{d\,u_r}{d\,t} = \sigma(x_r,u_r)\,p_r\,u_r\,x_r + k_r\,u_r + w_r \tag{10}$$

To demonstrate how these neuron models may exhibit emergent behaviour, two neurons are connected via an electrical synapse with a constant weight. Both neurons are driven by a controlled chaotic Rössler system stabilised into the same periodic orbit. Additionally, the first neuron receives periodic input of a square pulse at varying frequency. In the figures below, is shown the results of driving the mini-network with a period of 40 Hz and 33.3 Hz respectively. In all cases the chaotic control of the Rössler system is disabled at the beginning of the experiment to demonstrate the purely chaotic firing pattern and enabled at 500 ms. The control stabilises the system into a periodic orbit within a few timesteps. The periodic external pulse to the first neuron is enabled throughout.

With an external input period of 40 Hz the first neuron fires aperiodically before the control is enabled. After the control of the chaotic drive is enabled, the first neuron fires in a seemingly multi-orbit which is almost stable (figure 1*(a)*). However, the second neuron which has the same controlled chaotic drive as the first but receives input only from the first neuron ceases to fire (figure 1*(b)*). Changing the input frequency to 33.3 Hz, but leaving all else the same, the first neuron exhibits a clear multi-orbit after control is enabled (figure 1*(c)*) with a period of 1290.9 ms. The second neuron has a different orbit with a

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similar period but fires only three times in one period (figure 1*(d)*). Note that at 16.25 s, the second neuron fires four times but this is only a transient and it will settle into the three spike pattern at 2 s (not shown). Even though the neurons appear to be only semi-stable, in the sense that an element of noise or a transient element is present in the results, the different emergent behaviour of the second neuron is due to the response of the controlled chaotic neuron to the different external input frequencies when filtered by the first neuron.

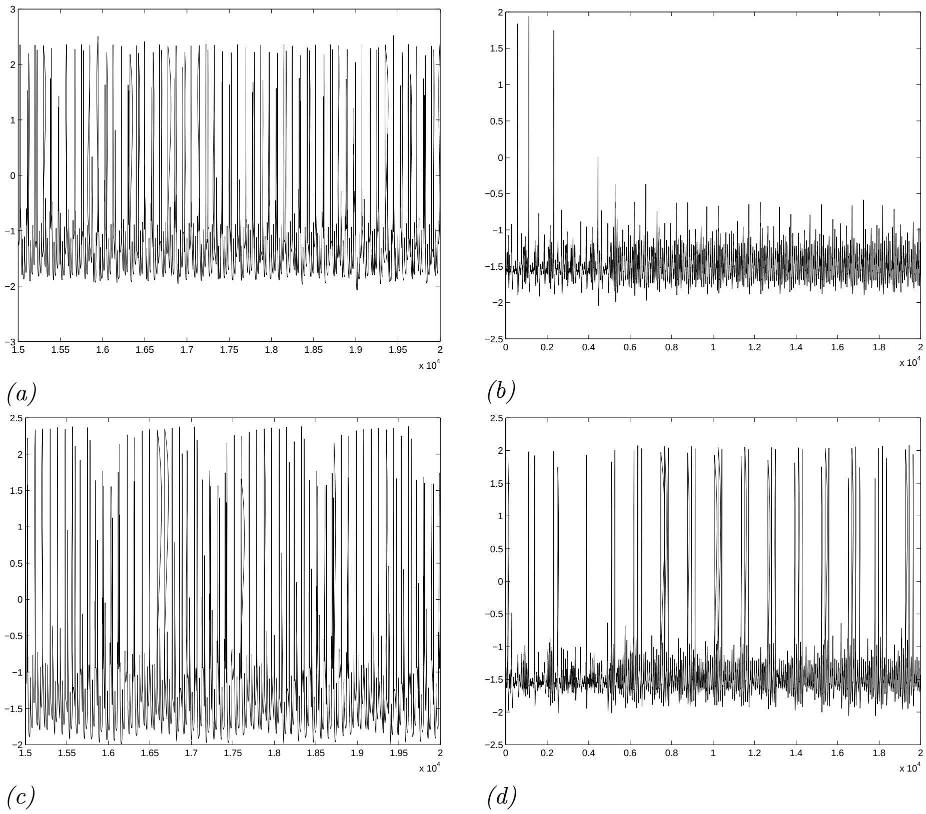

Fig. 1: *(a)* Voltage of the first neuron after chaotic control is enabled with external input of 40 Hz. *(b)* Voltage of the second neuron when control is enabled at 500 ms, input to the first neuron is 40 Hz. *(c)* Voltage of the first neuron after chaotic control is enabled with external input of 33.3 Hz. *(d)* Voltage of the second neuron after chaotic control is enabled with input to the first neuron of 33.3 Hz.

2.2 Membrane Computational Units

One aspect of neural modelling which has been considered to be less relevant to information processing is the signal conductance along the membrane. Using cable models and compartmental models, the possible unique properties of the membrane itself as computational unit are neglected. If we consider the membrane as a dynamic system with localised adaptation, we can formulate a membrane unit consisting of several components, such as ion channels and receptors, which together may act as a computational unit [18, 19]. With the aim of simulating computational processes within a membrane computational unit (MCU), we have build a phenomenological unit based on the Hindmarsh-Rose and Rössler models used above. Each model of an MCU has different components that may act together to produce a system which is capable of complex emergent behaviour. It generally consists of a spike generation component and an optional controlled chaotic drive component, i.e. an HR5 or HR4 model with or without R3 system.

By linking five computational units together a model may be built which synchronised two seperate inputs (SyncMCU). Two units, HR4R3-1 and 2, are made from four dimensional HR4 systems, driven by a controlled scaled Rössler system R3. Another unit, HR5-AND, consists of a single HR5 system, without a controlled chaotic drive, but electrically connected to units HR4R3-1 and 2. A fourth unit, HR4-ANDNOT, consists of a four dimensional HR4 system but with a scaled R3 drive. It receives input from units HR4R3-1 and 2. Lastly, the fifth unit, HR4, is a normal HR4 system without R3 drive, that only receives input from unit HR5-AND. All the R3 drive systems are controlled in the same unstable periodic orbit but the driving scalar is small such that by itself it does not cause the system to fire. The R3 systems may therefore act as a localised subcellular clock that can be in or out of sync with other units.

This configuration may act as a detector of desynchronisation of two input signals. Given an additional external input to the units HR4R3-1 and 2, which are combined in unit HR5-AND and then passed on to unit HR4, the unit HR4-ANDNOT will detect if unit HR4R3-2 fires but HR4R3-1 does not. Note that if they both fire, HR4-ANDNOT does not fire unless it has fired recently. We can now use this to attempt to synchronise unit HR4R3-2 with unit HR4R3-1 even if they have completely different periods.

To enable unit HR4-ANDNOT to synchronise the units HR4R3-1 and 2, a synchronisation function is defined as

$$\frac{dS}{dt} = \kappa_1(x_r^1 - x_r^2)\theta(x) - \kappa_2 S \tag{11}$$

where κ_1 and κ_2 are the growth and decay parameters, x_r^n are the x_r variables of the controlled chaotic scaled Rössler systems of the units that are synchronised. The function $\theta(x)$ is a threshold function on the x variable of the HR4 system of the unit HR4-ANDNOT. Parameters for (11) are $\kappa_1 = -0.75, \kappa_2 = 0.5$ with the threshold set at -0.5.

In the synchronised case, as shown in figures 2*(c)* and *(d)*, the emerging patterns are corrected by the synchronisation pulses HR4-ANDNOT unit and is much less noisy than in the unsynchronised case (not shown).