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Spicules-based competitive neural network

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Abstract. We present a new model of unsupervised competitive neural network, based on spicules. This model is capable of detecting topological information of an input space, determining its orientation and, in most case, its skeleton.

1 Introduction

Neural network models based on self-organizing maps, as proposed by Willshaw and von der Malsburg [1], Grossberg [2] and Kohonen [3], generate maps with a determinate dimension coming from an input space with larger dimensionality. These maps obtain a certain topological structure that preserves a neighborhood relation, which have the property of representing the highest density areas of input space. This property is interesting in different disciplines like voice recognition [4], combinatorial optimization [5], pattern recognition [6] and data compression [7].

The fact that similar maps have been found in human and animal brains, denotes that this topological preservation is important, at least in "natural processing systems".

In Kohonen's self-organizing maps, it is important to emphasize that both, the number of neurons and neighborhood topology between them, must be established before the learning process. This fact implies a great limitation, since in many cases there is not previous information that indicate us an adequate election of such parameters and forces to carry out, therefore, different simulations to determine them.

Another solution consists of determining these parameters incrementally during the learning process. This solution has been adopted and developed by diverse authors, proposing different models of neuronal networks included in what is generically named as "ontogenic networks" [8]. Within this classification we found the growing cell structures, developed by Fritzke [9], and the neuronal networks based on competitive hebbian learning, developed by Martinetz [10].

The growing cell structures [9] are able to generate dimensionality reducing mappings which may be used, for example, for visualization of high-dimensional data or for clustering. In contrast to Kohonen's self-organizing feature maps, which serves similar purposes, neither the number of units nor the exact topology has to be predefined in this model. Instead, a growth process successively inserts units and connections. Moreover, there is no need to define a priori the number of adaptation steps, making possible to continue the growth process until a specific network size or until an applications-dependent performance criterion is fulfilled.

The purpose of the competitive hebbian learning [10] is to distribute a number of centers according to some probability distribution (neural gas) and to generate a topology among these centers which has a dimensionality which is equal to the local dimensionality of the data and may be different in parts of the input space.

In this line, we present a model of competitive and unsupervised neural network based on spicules capable of detecting the topological information of an input space.

The remainder of this paper is organized as follows. In Section 2, we describe the segment based neural network, which is needed for the construction of the new model. In Section 3, we present the new spicules based competitive neural network. In sections 4, 5 and 6 the computation dynamics, the learning rule and the learning process of this network are respectively explained. In section 7 experimental results are shown and finally, in section 8, the conclusions of this work are presented.

2 The segment based neural network

The main contribution of the segment based neural network [11] is that the synaptic weights are composed of two reference vectors (dipoles), which determine the extremes of a segment. In other words, every unit of the network has associated a segment that will evolve in the learning process. Since the prototype of every unit is a segment, instead of a single vector, additional information, for example the predominant direction of the classes, can be obtained.

The topology of the network is similar to the topology of the simple competitive neural network: N input sensors and K neurons, where N is the dimension of input patterns. All input sensors are connected with all neurons.

The synaptic weight of every neuron in the network is determined by a pair of vectors ($\mathbf{w}_{i1}$, $\mathbf{w}_{i2}$). The notation is the following:

$$\mathbf{w}_{i1} = (w_{i1}^1, w_{i1}^2, w_{i1}^3, ..., w_{i1}^N)\ ,\ i = 1,2,\ldots K$$
$$\mathbf{w}_{i2} = (w_{i2}^1, w_{i2}^2, w_{i2}^3, ..., w_{i2}^N)\ ,\ i = 1,2,\ldots K$$

When an input pattern, $\mathbf{x}$, is presented to the network, the synaptic potential associated to the i-th neuron is

$$h_i = \alpha_i(\mathbf{x})h_{i1} + \overline{\alpha}_i(\mathbf{x})h_{i2} + \frac{1}{2}\alpha_i(\mathbf{x})\overline{\alpha}_i(\mathbf{x})\left\|\mathbf{w}_{i1} - \mathbf{w}_{i2}\right\|^2,\quad i = 1,2,\ldots,K \qquad (1)$$

where h_{i1} and h_{i2} are real numbers determined by the following expressions:

$$h_{i1} = \mathbf{w}_{i1}^T\mathbf{x} - \frac{1}{2}\mathbf{w}_{i1}^T\mathbf{w}_{i1} \text{ and } h_{i2} = \mathbf{w}_{i2}^T\mathbf{x} - \frac{1}{2}\mathbf{w}_{i2}^T\mathbf{w}_{i2},\quad i = 1, 2, ..., K$$

and

$$\alpha_i(\mathbf{x}) = \frac{(\mathbf{w}_{i1} - \mathbf{w}_{i2})^T(\mathbf{x} - \mathbf{w}_{i2})}{\left\|\mathbf{w}_{i1} - \mathbf{w}_{i2}\right\|^2}\ ,\quad \alpha_i(\mathbf{x}) + \overline{\alpha}_i(\mathbf{x}) = 1$$

As it is proof in [11], when an input pattern is presented to the network, only one unit is activated, the neuron with the highest synaptic potential, that is, the neuron whose segment is nearest to the input pattern $\mathbf{x}$. This neuron modifies its synaptic potential according to a learning rule obtained by gradient descent of a function that express the quadratic error of representing the input space by the structural segments of the network.

This model has been satisfactorily applied to classification problems [11] and to the skeleton detection of alphanumeric characters [12].

3 The spicules based competitive neural network

We name *spherule* to a structure constituted by a central point, called *nucleus*, and a set of segments, called *spicules*, that emanate from it. The nucleus has a synaptic weight denoted by $\mathbf{w_0}$ and the synaptic weights of peripheral points are denoted by $\mathbf{w_1}$, $\mathbf{w_2}$,…, $\mathbf{w}_L$, being L the number of spicules that constitute the spherule. An example of spherule with 3 spicules is shown in figure 1.

The topology of the network is similar to the topology of the simple competitive neural network: N input sensors and L+1 neurons, where N is the dimension of input patterns and L is the number of spicules of the spherule.

Formally, a spherule E with L spicules is defined by the following set:

$$E = \bigcup_{i=1}^{L} \left\{ e_i \in \Re^N : e_i = \alpha \cdot \mathbf{w}_0 + (\mathbf{1} - \alpha) \cdot \mathbf{w}_i , \forall \alpha \in [0,1] \right\}$$

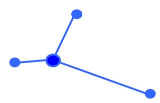

Fig. 1: Spherule with three spicules

Due to the fact that spicules are segments, the definition of the synaptic potential, h_i, of every spicule, e_i, is identical to the shown one in the expression (1). The distance D from an input pattern $\mathbf{x}$ to a spicule e_i is evaluated by the expression:

$$D(\mathbf{x}, e_i) = \|\mathbf{x} - e_i\| = \begin{cases} \|\mathbf{x} - (\alpha_i(\mathbf{x})\mathbf{w}_0 + (1 - \alpha_i(\mathbf{x}))\mathbf{w}_i)\| & \text{if} \quad \alpha_i(\mathbf{x}) \in (0,1) \\ \|\mathbf{x} - \mathbf{w}_0\| & \text{if} \quad \alpha_i(\mathbf{x}) \geq 1 \\ \|\mathbf{x} - \mathbf{w}_i\| & \text{if} \quad \alpha_i(\mathbf{x}) \leq 0 \end{cases}$$

where

$$\alpha_i(\mathbf{x}) = \frac{(\mathbf{w}_0 - \mathbf{w}_i)^T (\mathbf{x} - \mathbf{w}_i)}{\|\mathbf{w}_0 - \mathbf{w}_i\|^2}$$

Finally, the distance between an input pattern $\mathbf{x}$ and a spherule E is given by

$$D(\mathbf{x}, E) = \|\mathbf{x} - E\| = \min_{1 \leq i \leq L} \|\mathbf{x} - e_i\|$$

4 Computation dynamics

When an input pattern $\mathbf{x}$ is presented to the network, only one spicule is activated, the spicule e_r that is nearest to the input, satisfying the following expression:

$$\|\mathbf{x} - e_r\| = \min_{1 \leq i \leq L} \|\mathbf{x} - e_i\|$$

In this sense, the spicule activated is the one with the highest synaptic potential, as is proved by the following theorem:

Theorem 1.

$$h_r > h_i \;\Leftrightarrow\; \|\mathbf{x} - e_r\| < \|\mathbf{x} - e_i\| \;,\;\; \forall i \neq r \;,\;\; i = 1,2,...,L$$

Proof of Theorem

Due to the fact that spicules are segments, the proof is identical to the theorem proposed in [11].

5 The learning rule

Let us consider an input space constituted by p patterns (vectors). For the determination of the learning rule, we used the principle of the minimum empirical risk, given by the following distortion function:

$$E_{distorsion} = \sum_{\mu=1}^{p}\sum_{i=1}^{L} M_i^{\mu}(\mathbf{x})\left\|\mathbf{x}^{\mu} - e_i\right\|^2 \;,\;\; M_i^{\mu}(\mathbf{x}) = \begin{cases} 1 & \text{if} \quad \|\mathbf{x} - e_i\| = \min\limits_{j=1,2,...,L}\|\mathbf{x} - e_j\| \\ 0 & \text{in another case} \end{cases} \qquad (2)$$

where

$$e_i = \left\{e \in \Re^N : e = \alpha \cdot \mathbf{w}_0 + (\mathbf{1} - \alpha) \cdot \mathbf{w}_i , \forall \alpha \in [0,1]\right\} \;,\quad i = 1,2,...,L$$

That is, the problem consists of determining the L spicules of the spherule that diminish the distortion error expressed in (2). The learning rule is deduced by the gradient descent method of a distortion function similar to expression (2), obtaining the following:

$$\begin{aligned} &\text{- if } M_i^{\mu}(\mathbf{x}) = 1 \qquad && \mathbf{w}_0(k+1) = \eta(k)\cdot\mathbf{x}(k) + (1-\eta(k))\cdot\mathbf{w}_0(k) \\ & && \mathbf{w}_i(k+1) = \eta(k)\cdot\mathbf{x}(k) + (1-\eta(k))\cdot\mathbf{w}_i(k) \\ &\text{- if } M_i^{\mu}(\mathbf{x}) = 0 && \mathbf{w}_i(k+1) = \mathbf{w}_i(k) \end{aligned} \qquad (3)$$

where k is the iteration in course, and $\eta(k)$ is the learning parameter. This parameter is a monotonous decreasing function that depends on the number of iterations. Let us consider $\eta(k)$ as

$$\eta(k) = \eta_0(1 - k/T) \;,\;\; k = 1,2,...,T$$

being T the total number of iterations and $\eta_0 \in [0,1]$ the initial value.

From the learning rule exposed in (3) it can be deduced that, when an input pattern **x** is presented to the network, only the end of the winning spicule is modified, but, since the nucleus of the spherule is common to all the spicules, it will be always modified in every step of the learning process.

6 The learning process

Similarly to Kohonen's self-organizing maps [3], the learning process is divided in two phases: the *ordering phase* and the *convergence phase*:

- Ordering phase: during this phase the adaptive process takes place in which the nucleus of the spherule is positioned in the gravity centre of input space and spicules are distributed according to the topological structure of this input space. As much the synaptic weights of the nucleus as all the weights of the spicules ends, are modified with the same learning parameter, $\eta(k)$.

- Convergence phase: this phase is needed to fine-tune the spherule structure and, therefore, provide an accurate statistical quantification of the input space. The synaptic weights are modified with two different learning parameters: $\eta_a(k)$ for the synaptic weight of the nucleus and $\eta_b(k)$ for the synaptic weights of the spicules ends.

In the ordering phase, the initial value for the learning parameter, η_0, will be near to one whereas in the convergence phase, the initial values for the learning parameters, η_{a0} and η_{b0}, will be inferior that the previous one, fulfilling in addition that $\eta_{b0} \geq \eta_{a0}$. Moreover, the number of necessary iterations for the convergence phase must be minor that in the ordering phase.

7 Experimental results

Different shapes in digital images have been considered using, for all of them, the same configuration: in order to obtain the input patterns, 800 points have been randomly chosen within such figures (p = 800); the ordering phase has 4 iterations with η_0 = 0.9 and the convergence phase has 2 iterations with η_{a0} = 0.1 and η_{b0} = 0.3.

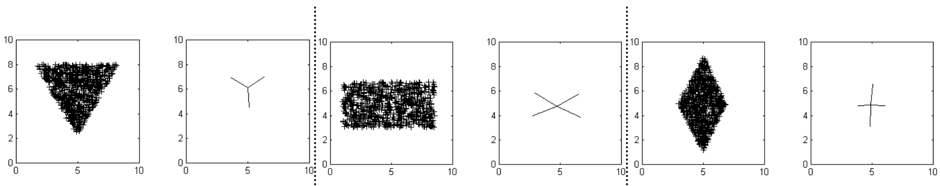

Fig. 2: Results from three different convex shapes

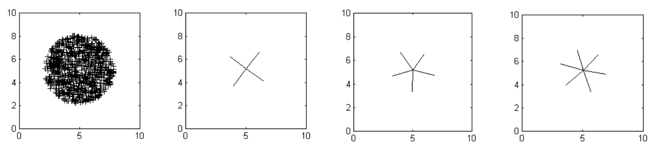

Fig. 3: Results from a circle shape with different number of spicules

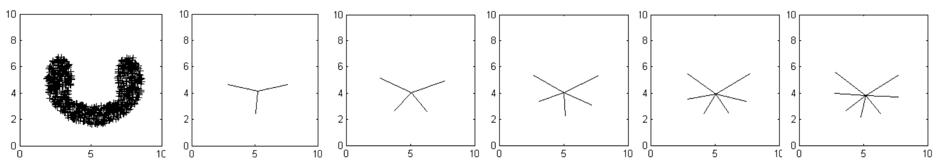

Fig. 4: Results from a non-convex shape with different number of spicules

In figure 2, the result of the network is shown from three different convex shapes (a spherule with 3 spicules for the triangle and with 4 spicules for the rectangle and rhombus). It can be appreciated that spicules are topologically organized, allowing that the resulting spherule constitutes the skeleton of the different input spaces and, therefore, giving very valuable information that could be used for a possible automatic recognition of shapes. In figure 3, the results of the network from a circle shape with different number of spicules are shown. An interesting result can be appreciated, as spicules are added to the spherule, these are organized radially, so that the circle is partitioned in portions of similar size. In figure 4, the results from a non-convex shape are shown. As spicules are added to the network, these are distributed along the shape, looking for its topological characteristics.

In all simulations, it can be observed that the nucleus of spherule is positioned in the gravity centre of input spaces, but not necessarily over input patterns like in figure 4, whereas spicules ends are always distributed over them.

8 Conclusions

We present a new model of unsupervised competitive neural network based on spicules in which a new hierarchic structure constituted by association of several segments is developed. This hierarchic structure is called *spherule* and it is constituted by a central point, called *nucleus*, and a set of segments, called *spicules*, that emanate from it. As it is presented in experimental results, this model is capable of detecting topological information of an input space, determining its orientation and, in most case, its skeleton.

At the moment, we are working in providing to the network, on the one hand, with a mechanism that allows to identify the optimal number of spicules and, by another one, of a process of replication or multiplication of spherules in the assumption in which the input space is constituted by more than one group.

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