

## SOM based clustering with instance-level constraints

Fazia Bellal<sup>1</sup>, Khalid Benabdeslem<sup>1</sup> and Alexandre Aussem<sup>1</sup>

1- University of Lyon1, LIESP  
8, Avenue Niels Bohr, 69622 Villeurbanne cedex, France

### Abstract.

This paper describes a new topological map dedicated to clustering under instance-level constraints. In general, traditional clustering is used in an unsupervised manner. However, in some cases, background information about the problem domain is available or imposed in the form of constraints, in addition to data instances. In this context, we modify the popular SOM algorithm to take these constraints into account during the construction of the topology. We present experiments on synthetic known databases with artificial constraints. We then apply the new method to a real problem of clustering melanoma data in health domain.

## 1 Introduction

Knowledge extraction by data mining often involves unsupervised clustering processes so as to optimize the organizing of data sets in regard of their similarities. However, these algorithms only access to variables which describe each data but they do not deal with any other kind of given information. Nevertheless, taking *a priori* knowledge into account in such algorithms, if there exists, is an important problem concerning at the same time the expression, structuration and formalisation of knowledge in view of its integration in automatic clustering processes. The first work on this area was presented in [6] based on the modification of the COBWEB algorithm proposed in [3]. Furthermore, the same authors have proposed another approach which integrates constraints in the  $K$ -means algorithm [7]. Their COP- $k$ -means algorithm attempts to find a set partition that minimizes the vector quantization error and satisfies all constraints at the same time. Moreover, in [9], an exploration of the use of instance and cluster-level constraints was performed with agglomerative hierarchical clustering. In [2] it is shown improvements in quality and computational complexity of clustering by using constraints in a graph b-coloring clustering algorithm. The authors empirically illustrate the benefits from using constraints to improve cluster purity and average distortion. Alternatively, in this paper, we propose a new version of Kohonen's algorithm based on the control of neurons respecting or violating some given constraints on patterns during the construction of the topological map. Thereby, we will compare our method to all cited constrained-clustering based methods.

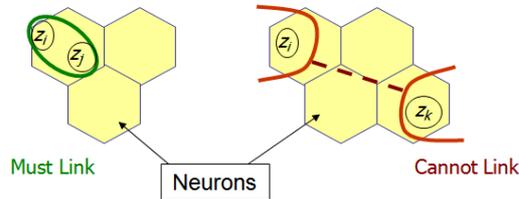


Fig. 1: Must-Link and Cannot-Link constraints

## 2 Constrained Topological Map

### 2.1 Notations

Let us assume that a set of  $N$  samples or patterns  $z_i = (z_{i1}, \dots, z_{id}) \in \mathbb{R}^d$  is given where  $d$  is the number of variables. The learning database formed by these samples is denoted by  $A = \{z_i \in \mathbb{R}^d, i = 1, \dots, N\}$ . We also suppose that we have an undirected graph  $G = (V, E)$  where  $V$  is a set of vertices and  $E$  a set of edges. This graph represents a neural network by considering that each vertex  $v \in V$  is a unit or neuron. The number of neurons in  $V$  is denoted by  $C$ . The distance  $\delta(c, r)$  between two neurons  $c, r \in V$  is the length of the minimal path linking  $c$  and  $r$  in  $G$ . In order to take into account the influence of a neuron  $r$  on a neuron  $c$ , we introduce a symmetric kernel function  $\mathcal{K} : \mathbb{R} \rightarrow \mathbb{R}_+$  satisfying

$$\lim_{|x| \rightarrow \infty} \mathcal{K}(x) = 0, \quad (1)$$

so that the mutual influence between  $r$  and  $c$  is  $\mathcal{K}(\delta(c, r))$ . The neighborhood set around neuron  $c$  is denoted  $Neighbor(c)$ . It contains all nodes up to a certain radius  $T$  in the grid from node  $c$ . For each neuron  $c \in V$ , a reference vector  $w_c^t = (w_{c1}^t, \dots, w_{cd}^t) \in \mathbb{R}^d$  is defined where  $0 \leq t \leq N_{iter}$ ,  $N_{iter}$  being the maximum number of iterations in Kohonen's algorithm [4].

### 2.2 Generation of constraints

In some situations, much unlabeled data and a little labeled data are available in the dataset. Thereby, we express this knowledge in the form of two types of constraints (see fig. 1) :

- **Must-Link constraints**, denoted by  $Con_{=}(z_i, z_j)$  indicate that two patterns  $z_i$  and  $z_j$  must belong to the same neuron,
- **Cannot-Link constraints**, denoted by  $Con_{\neq}(z_i, z_j)$  indicate that two patterns  $z_i$  and  $z_j$  cannot belong to the same neuron.

It appears that *Must-Link* constraints are transitive, i.e., for any patterns  $(z_i, z_j, z_k) \in V$ , if  $Con_{=}(z_i, z_j)$  and  $Con_{=}(z_j, z_k)$  are true, then it follows that  $Con_{=}(z_i, z_k)$  must also be satisfied. Similarly, if  $Con_{=}(z_i, z_j)$ ,  $Con_{=}(z_k, z_l)$  and  $Con_{\neq}(z_i, z_k)$  are true, then  $Con_{\neq}(z_i, z_l)$  and  $Con_{\neq}(z_j, z_k)$  are also satisfied.

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**Algorithm 1** RespectCon( $A, V, i, Con_=, Con\neq$ )

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1:  $V_i^{viol} = \emptyset$ 
2: for all  $c \in V$  do
3:   for all  $z_j$  such that  $Con_=(z_i, z_j)$  do
4:     if  $z_j \notin c$  then  $V_i^{viol} = V_i^{viol} \cup \{c\}$ 
5:   end for
6:   for all  $z_k$  such that  $Con\neq(z_i, z_k)$  do
7:     if  $z_k \in c$  then  $V_i^{viol} = V_i^{viol} \cup \{c, Neighbor(c)\}$ 
8:   end for
9: end for
10:  $V_i^{resp} = V \setminus V_i^{viol}$ 
11: return  $V_i^{resp}$ 

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We define  $V_i^{resp} \subset V$  as the set of all neurons in  $V$  satisfying constraints on a given pattern  $z_i \in A$ . In the same way,  $V_{viol} \subset V$  is the set of all neurons in violation with  $z_i \in A$ . In the case of *Cannot-Link* constraints, the idea is to clamp neurons which are located in the neighborhood set around the neuron violating these constraints. The computation of  $V_i^{resp}$  is performed by removing from  $V$  neurons which violate constraints on  $z_i$  as described in algorithm 1. Patterns  $z_i$  which over-constrain the problem are omitted.

The computation of sets  $V_i^{resp}$  for all patterns  $z_i \in A$  is the key of our approach since it will allow the integration of constraints in Kohonen's algorithm.

### 2.3 The constrained algorithm

Let us assume that  $t > 0$  and that a pattern  $z_i$  has been randomly chosen in  $A$ . Contrary to Kohonen's approach [4], the best-matching unit  $c^*$  is now sought in the subset  $V_i^{resp} \subset V$  computed from algorithm 1. In other words, we have:

$$c^* = \arg \min_{c \in V_i^{resp}} \|z_i - w_c^t\|^2. \quad (2)$$

Moreover, only neurons influenced by constraints on  $z_i$  may be corrected after the computation of  $c^*$ . In other words, the stochastic gradient based correction scheme is now restricted to  $V_i^{resp}$ :

$$w_c^t = w_c^{t-1} - \mu^t \mathcal{K}^T(\delta(c, c^*))(w_c^{t-1} - z_i), \quad \forall c \in V_i^{resp}, \quad (3)$$

where  $\mu^t$  is the learning rate which is a decreasing function of time. These modifications lead to a new algorithm called *CrTM*.

## 3 Experimental results

### 3.1 Evaluation metrics

For the evaluation of the efficiency of the CrTM algorithm, we propose to use the Rand index [5]. This index measures the correspondance between two partitions

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**Algorithm 2** CrTM( $A, V, Con_=, Con\neq, N_{iter}$ )

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- 1: Set  $t = 0$  and initialize vectors  $w_c^t$  to random values for all  $c \in V$ .
  - 2: **for**  $t = 1, \dots, N_{iter}$  **do**
  - 3:   Select a random pattern  $z_i \in A$
  - 4:   Compute  $V_i^{resp} = RespectCon(A, V, i, Con_=, Con\neq)$
  - 5:   Find  $c^* \in V_i^{resp}$  as defined in (2).
  - 6:   Update reference vectors using (3).
  - 7: **end for**
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$P_1$  and  $P_2$  of a data set  $A$ . In our case,  $P_1$  is the correct partition produced by labels of predefined classes and  $P_2$  is the partition obtained from the *CrTM* algorithm. Each partition is regarded as a set of  $N(N-1)/2$  pairs of decisions. For each pair of points  $(z_i, z_j)$ ,  $P_i$  assigns them to the same class or to two different classes. Assuming  $a$  is the number of decisions where  $z_i$  belongs to the same class as  $z_j$  in  $P_1$  and  $P_2$  and  $b$  is the number of decisions where  $z_i$  and  $z_j$  do not belong to the same class in  $P_1$  and  $P_2$ , we obtain  $(a+b)$  correct decisions and the *overall accuracy* between  $P_1$  and  $P_2$  is:

$$Acc = Rand(P_1, P_2) = \frac{a+b}{N(N-1)/2}. \quad (4)$$

We also show that knowledge brought by constraints may even improve the performance of classifiers on patterns which are not constrained. Then, we compute aside *overall accuracy*, the one on a *held-out test set* which is a subset of data set composed of instances that are not directly or transitively affected by the constraints. This represents a real learning performance measure since such a *Held-Out* improvement reveals if the algorithm managed to learn constraints and to generalize this type of knowledge so as to influence the classification of unconstrained patterns.

### 3.2 Results obtained with artificial constraints

We propose to evaluate the CrTM algorithm on two labeled databases issued from the UCI bank [1]: *Tic-tac-toe* and *Heart disease*. To generate artificial constraints, we randomly select two instances from the data set and check their labels. If they have the same label, we create a *Must-Link* constraint, otherwise, a *Cannot-Link* constraint. We compare our algorithm using results obtained on these databases to three other constrained methods. We use a random initialization, a gaussian neighborhood function and default values for the learning parameters. The database *Tic-tac-toe* contains 100 patterns described by 9 qualitative variables and 2 output classes. Without any constraint, the overall accuracy obtained with Kohonen's algorithm is 66.6%. Defining 500 random constraints, we obtain 96.7% for overall accuracy and 91.4% for held-out as illustrated in figure 2. With the same number of constraints, the results obtained with COP-COPWEB, COP-Kmeans and COP-b-coloring are 49%, 56%

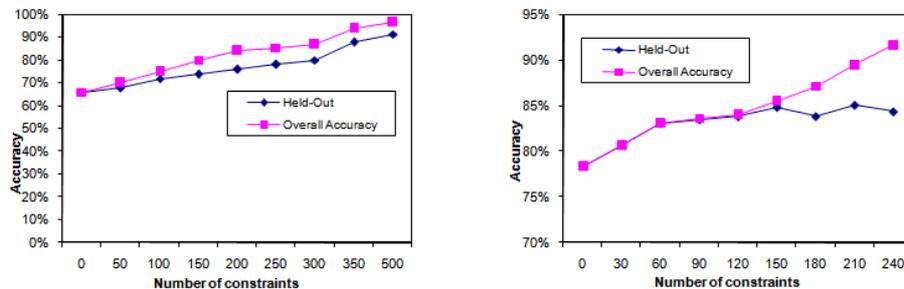


Fig. 2: Results obtained with the CrTM algorithm on *Tic-tac-toe* (left) and *Heart disease* (right)

and 82% respectively [6, 7, 2]. Thus, the CrTM algorithm yield a significant improvement. The database *Heart disease* contains 303 patterns described by 5 numerical variables and 8 qualitative variables. The instances were also classified into two classes. Without any constraint, the overall accuracy for Kohonen's algorithm is 78%. Using 240 random constraints, the overall accuracy for CrTM rises to 91% (see fig. 2). Using 150 random constraints, held-out reaches 85%. In comparison, the overall accuracy for COP-b-coloring is 50% without constraints and the held out is 66% with 500 constraints. On the one hand, we observe that constraints always improve the quality of the partition, and on the other hand we note that our algorithm gives a better clustering than others with a smaller number of constraints.

### 3.3 Application to real data : Melanoma cancer

Melanoma is a malignant tumor of melanocytes which are found predominantly in skin but also in the bowel and the eye. The melanoma base used in this study, was kindly supplied by *VISOON* company, Lyon, France. It contains 226 pictures of melanoma. Each picture is described by the "ABCD rule" [8] which is a widely accepted tool to promote early detection of melanoma: **A**symmetry, **B**order irregularity, **C**olor variegation, and **D**iameter greater than 6 mm. Doctors labelled some picture of data, that is to say these patterns are associated with an output class concerning the disease statement (either healthy or with heart-disease). Therefore, we generate real constraints imposed by experts. The overall accuracy obtained with Kohonen's algorithm is 78.2% without constraints. Adding 210 constraints to the problem, the overall accuracy for CrTM is 84%, and the held-out is 81.2% as illustrated in fig.3.

## 4 Conclusion

In this paper we proposed a general method for incorporating background knowledge during the construction process of self-organizing maps. We transform background knowledge, from partially labeled data or a priori knowledge on the

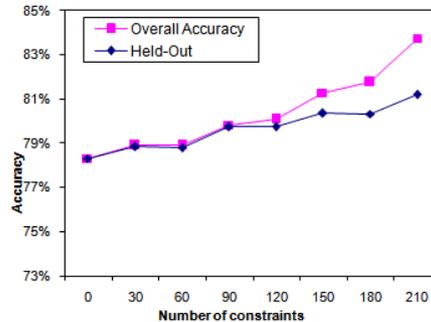


Fig. 3: Performance of CrTM on melanoma data

domain of real data sets, in the form of instance-level constraints. The proposed algorithm deals with binary (between two pair of instances) and deterministic (belonging or not to the same cluster) constraints. Our experimental results indicate that only a small amount of constraints is necessary to significantly improve the quality of the clustering. Furthermore, the empirical study we have carried out tends to demonstrate that prior information may be applied in a real domain. The future of our work will concern the extension of our method to more complex constraints such as probabilistic or conditional constraints and the optimization of the CrTM algorithm using a hierarchical clustering.

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