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A Wavelet-heterogeneous Index of Market Shocks for assessing the Magnitude of Financial Crises

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Abstract. An accurate quantitative definition of financial crisis requires a universal and robust scale for measuring market shocks. Following Zumbach *et al.* (2000) and Maillet et Michel (2003), we propose a new quantitative measure of financial disturbances, which captures the heterogeneity of investor horizons – from day traders to pension funds. The indicator resides on a multi-resolution analysis of market volatility, each scale corresponding to various investment horizons and different data frequencies. This new risk measure, called "Wavelet-heterogeneous Index of Market Shocks" (WhIMS), is based on the combination of two methods: the Wavelet Packets Sub-band Decomposition and the constrained Independent Component Analysis (See Kopriva and Seršić, 2007 and Lu and Rajapakse, 2005). We apply this measure on the French stock markets (high frequency CAC40) to date and gauge the severity of financial crises.

1 Introduction

A clear understanding of financial stability requires an accurate measure of financial turbulences. It is important to know when such pressure occurs and what its intensity is to detect, identify and compare the severity of different crises. The objective of this paper is to propose a new quantitative measure of financial crises, which we called the WhIMS (for Wavelet-heterogeneous Index of Market Shocks). The WhIMS is computed in two steps. The first one is a combination of Wavelet Packets and a Subband Decomposition Independent

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Component Analysis (See [4] and [5]). The difference with the traditional ICA is that this method allows us to get components truly independent and to introduce a constraint in the signals to reduce the data dimension. Moreover, this tool is more appropriate than the Principal Component Analysis when data are non linear and non Gaussian. The second step consists in fitting cumulative density function of Independent Components with a Generalized Pareto Distribution (GPD) based on L-moments estimation method (which are order-statistic cumulants). Indeed, this step guarantees the WhIMS not to depend anymore on the hypothesis of Log-normality of volatilities.

The measure of market disturbances we propose is a refinement of the so-called Scale of Market Shocks (SMS) by Zumbach *et al.* [8] and the Index of Market Shocks (IMS) by Maillet and Michel [6]. Indeed, the WhIMS would overcome few shortcomings of the IMS recognized by their authors. First, the method of estimating volatilities at different scales by varying sampling frequency can lead to heterogeneous estimation errors and incomparable estimates. Second, the definition of independent components based on linear correlations is too restrictive when factors are not symmetrically distributed.

The motivation behind the wavelet decomposition is the existence of traders with different time horizons (see e.g. [2]). Short-term traders, such as day-traders, are constantly watching the market; they re-evaluate the situation and execute transactions at a high frequency horizon. Long-term investors, such as pension funds, may look at the market less frequently. Sometimes, price movements may have a certain influence on the timing of both day-traders and long-term investors'transactions and investment decisions. The WhIMS gauges the severity of these markets fluctuations that impact both day-traders, mutual funds and pension funds.

The outline of this paper is as follows. Section 2 introduces the Independent Component Analysis and the Wavelet Packet Sub-band Decomposition. Section 3 presents the Generalized Pareto Distribution estimated based on L-moments estimation method. Section 4 explains the WhIMS computation. Section 5 presents empirical estimations.

2 Scale-by-scale Decomposition of Volatility

The method of Independent Component Analysis (ICA) is well-known in Signal Processing and applied in various field such as biomedicine, speech and telecommunication signals. It is the most used method for Blind Source Separation (BSS). However, the mobilization of the ICA leads to several limits. Indeed, Chang and Zhang [1] determine that the independence property of the extracting independent components is not always verified which could bias our results. We combine two decomposition and factor analysis methods: the Wavelet Packet Sub-band Independent Composition Analysis (WPSD-ICA) and the constrained Independent Component Analysis (cICA). The result of this mix gives the WPSD-cICA.

Our approach to multi-scale analysis is based on the wavelet transform of

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the original time series of stock returns. The definition of WhIMS resides on the estimation of the vector of volatilities at different scales $\sigma^{(1)}, ..., \sigma^{(m)}$. A natural tool for this purpose is wavelet analysis. It is widely used in signal processing in order to decompose a given time series $x(t)$ called “signal” into a hierarchical set of approximations and details (multi-resolution analysis), and to decompose energy of the signal on scale-by-scale basis.

Any decomposition is based on the so-called wavelet mother function Ψ and the associated wavelet father function Φ. Ψ is used to define the details while Φ is used to define approximations. Then, the wavelet transform C consists in calculating a ”resemblance index” between the signal and the wavelet function located at position b and of scale a :

$$C(a,b) = \int_{-\infty}^{+\infty} x(t) \frac{1}{\sqrt{a}} \Psi\left(\frac{t-b}{a}\right) dt. \quad (1)$$

This transformation can be applied at continuous or discrete scales. For a properly chosen wavelet function, the synthesis of the original signal can be done:

$$x(t) = \sum_{j=1}^{N} \sum_{k=1}^{K} C(j,k) \Psi_{j,k}(t), \quad (2)$$

with j corresponding to decomposition levels and k to positions of wavelet on the time axis.

3 Estimation of a GPD with a Generalized Method of TL-moments

Recent attempts for modelling distributions in a multivariate framework are built on the order-statistics using L-moments. One of their main advantage over the conventional (C-)moments is that their empirical counterparts are less sensitive to the effects of sampling variability. They are shown to provide more robust estimators of higher moments that the traditional sample moments and have then found wide applications in fields where extreme events matter, such as meteorology, hydrology and also earthquake analysis with the Richter Scale [7]. More precisely, L-moments are defined as certain linear functions of the Probability Weighted Moments and can characterize a wider range of distributions compared to the usual moments.

To compute the WhIMS, we uses a GPD estimated with Generalized Method of TL-moments [3]. The distribution of a GPD is defined by three parameters: $\upsilon \in I\!R$, the location parameter, $\alpha \in I\!R_+$, the scale parameter and $\xi \in I\!R$ the tail index. The GPD distribution of the WhIMS block *maxima*, denoted here $\widehat{\sigma}$ for the sake of simplicity, is given by:

$$G_\xi(\widehat{\sigma}) = \begin{cases} 1 - \left[1 + \xi \frac{(\widehat{\sigma}-\upsilon)}{\alpha}\right]^{-\xi^{-1}} & \text{if } \xi \neq 0 \\ 1 - Exp\left[-\frac{(\widehat{\sigma}-\upsilon)}{\alpha}\right] & \text{otherwise,} \end{cases}$$

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for every $\widehat{\sigma} \in \mathcal{D}_2$, defined by:

$$\mathcal{D}_2 = \begin{cases}]-\infty; \upsilon - \frac{\alpha}{\xi}[& \text{if } \xi < 0 \\ I\!R & \text{if } \xi = 0 \\ ]\upsilon - \frac{\alpha}{\xi}; +\infty[& \text{if } \xi > 0. \end{cases}$$

This allows us to deduce the cumulants of order 3 and 4 for a GPD normalized distribution:

$$\begin{cases} \kappa_3 = \frac{2\alpha^3(1+\xi)}{(1-\xi)^3(1-2\xi)(1-3\xi)} \\ \kappa_4 = \frac{3\alpha^4(3+\xi+2\xi^2)}{(1-\xi)^4(1-2\xi)(1-3\xi)(1-4\xi)} - 3. \end{cases} \quad (3)$$

Moreover, the first three TL-moments, as a function of the three characteristic parameters of a GPD distribution, are given, for every $(s,t) \in I\!N^2$, by:

$$\begin{cases} \lambda_1^{(s,t)} = & \upsilon - \frac{\xi}{\alpha} + \frac{(1+s+t)!}{t!} \frac{\Gamma(t-\xi+1)}{\Gamma(2+s+t-\xi)} \frac{\xi}{\alpha} \\ \lambda_2^{(s,t)} = & \frac{(2+s+t)!}{2(t+1)!} \frac{\Gamma(t-\xi+1)}{\Gamma(3+s+t-\xi)} \alpha, \\ \lambda_3^{(s,t)} = & \frac{(3+s+t)!}{3(t+2)!} \frac{\Gamma(t-\xi+1)}{\Gamma(4+s+t-\xi)} (1+\xi)\alpha, \end{cases} \quad (4)$$

where $\lambda_r^{(s,t)}$ is the r-th TL-moment of truncation order (s,t), $\upsilon \in I\!R$ the location parameter, $\alpha \in I\!R_+$ the scale parameter, $\xi \in I\!R$ the tail index and $\Gamma(a) = \int_0^{+\infty} t^{a-1} e^{-t} dt$ is the Gamma function.

4 The Wavelet-heterogeneous Index of Market Shocks (WhIMS) Computation

The WhIMS *formula* is:

$$WhIMS_t = -\alpha \sum_{i=1}^{K} \left\{ w_k \log_2 \left[1 - F\left(\sigma_i^2\right) \right] \right\} \quad (5)$$

where $F(.)$is the cumulative density function, σ_i represents each independent factors of volatilities at different time scales, w_k is the weight of each Independent Components obtain from a Wavelet Packets Sub-band Decomposition constrained Independent Component Analysis, and α a scaling constant.

Finally, the algorithm of computing WhIMS can be summarized as follows. Firstly, on a moving window of fixed length (equal to a power of two) perform the Wavelet Packet Subband Decomposition of returns; secondly, we reconstruct the trajectories of returns for each scale; thirdly, we transform data for all scales to log-squared returns; then, we compute constrained Independent Components and their weights; next, we fit the generalized Pareto distribution for the tails of each Independent Component; and finally, we compute the WhIMS using *formula* (5);

The next section contains empirical results obtained for the French stock market data.

5 Empirical Results

We estimate the WhIMS using CAC40 high frequency (15 minutes intervals) data between 1997 and 2007. This period takes account of two financial major events: the Asian crises of 1998 and the crises of the technological values during 2000 until 2003. Below, you can see an illustration of our data. After decomposing the source signal, we build new trajectories of returns at different time scales. Each level of decomposition j corresponds to a frequency of 2^j days. Our objective is to identify regimes of financial crises that are defined when our risk measure exceeds the arbitrary threshold corresponding to a 90% confidence level. Next, we applied the WhIMS algorithm to our data in order to detect and identify market turbulences during the studying period using the *formula* (5). Figure 1 shows the evolution of the CAC40 index and the computed WhIMS. The WhIMS appears relatively temperate during the period 2000-2003 and the "death of the volatility", with a rebirth of volatility in the recent months corresponding to the latest credit events.

6 Conclusion

We proposed a revised version of the Index of Market Shocks, based on a multi-resolution analysis of market volatility. The indicator aims to characterize volatility as perceived by different types of market agents who have various investment horizons. It can be computed both for large samples of low-frequency data and high frequency data. The algorithm of computing WhIMS is based on wavelet decomposition combined with a factor analysis, introduced as tools to decompose volatility in both time and scale and to identify volatility factors at each scale. The WhIMS does not rely on the log-normality of these factors, making it more robust to distributional properties of the data. We established a quantitative definition of crises based on the distribution of the WhIMS to date events on financial markets, as well as to compare the relative severity of these crises. Further researches could extend the WhIMS to different markets measuring contagion or interdependence between them and justify theoretically the multi-resolution analysis of market volatility.

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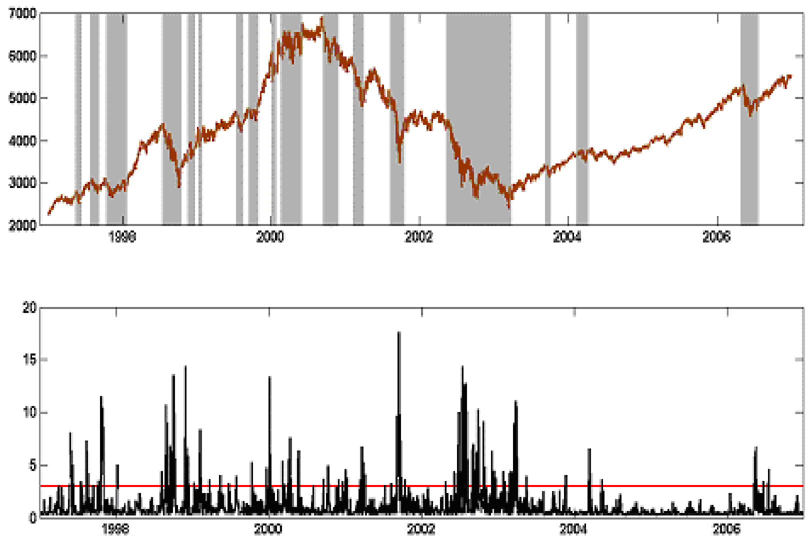
Fig. 1: Evolution of the CAC40 Index and the Wavelet-heterogeneous Index of Market Shocks



Sources: Euronext. CAC40 30 minutes intraday data during January 2nd, 1997 to December 31st, 2007. The chart on the top represents the evolution of the CAC40 in brown and crises in gray (WhIMS superior to the threshold of 3 which corresponds to the 90% highest values of the WhIMS). The chart on the bottom represents the WhIMS applied to CAC40 in black line and the 90% threshold in red line. Computation by the authors.