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A Robust Hybrid DHMM-MLP Modelling of Financial Crises measured by the WhIMS

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Abstract. This paper develops a hybrid model combining a Hidden Markov Chain (HMC) and Multilayer Perceptrons (MLP) on the Wavelet-heterogeneous Index of Market Shocks (WhIMS) to identify dynamically regimes in financial turbulences. The WhIMS is an aggregate measure of volatility computed at different frequencies. We estimate the model based on a French market stock index (CAC40 Index) and compare the prediction performance of the HMC-MLP model to classical linear and non-linear models. A state separation of financial disturbances based on the WhIMS and conditional probabilities of the HMC-MLP model is then performed using a Robust SOM.

1 Introduction

Financial markets occasionally experience sudden and large falls in asset prices. Such extreme events constitute a severe threat for the stability of the financial system and international investors not only with the large losses they cause but also with the deterioration of the risk-return trade-off and the increase of conditional correlation and volatility across the markets.

Following Maillet *et al.* [7], this paper proposes to identify regimes in financial turbulences, i.e. normal and crisis states. We present a model for the Wavelet-heterogeneous Index of Market Shocks (WhIMS) based on a Hidden Markov Chain (HMC) and Multilayer Perceptrons (MLP). The WhIMS is an easily computable measure that quantifies the intensity of market movements. In short, its construction is based on an analogy with the so-called Richter scale used for measuring earthquake intensity. The main step consists in applying a Wavelet Packets Sub-band Decomposition constrained Independent Component Analysis (WPSD-cICA) first to decompose the return volatility at different time scales and, secondly, to extract independent factors resulting of the decomposition. Then, we fit volatility extreme values from a Generalized Extreme Value

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(GEV) based on the L-moment method. The WhIMS is a refinement of the so-called Index of Market Shocks (IMS) by Maillet and Michel [6]. In brief, contrary to the IMS, the WhIMS accounts for the non-normality and the non-linearity of data and does not to depend on the hypothesis of Log-normality of volatilities.

We first present the Autoregressive Hidden Markov Chain Models. We estimate the model based on a French stock market index (CAC40 Index) and compare the prediction performance of the HMC-MLP model to classical linear and non-linear models. A state separation of financial disturbances based on the WhIMS and conditional probabilities of the HMC-MLP model is finally performed using a Robust SOM.

2 AutoRegressive Hidden Markov Chain Models

Over long periods, financial time series exhibit important breaks in their behavior and properties. Abrupt changes may be due to several reasons, such as bankruptcies, burst of bubbles, structural changes in business cycles conditions (*i.e.* disinflation), or wars and related events. One way to capture structural breaks is to use hidden Markov chains. The hidden Markov chain , initially proposed by Baum and Petrie [1], has been widely applied in various fields, including speech recognition, signal processing, DNA recognition and financial time series.

Let us consider $(Y_t)_{t\in\mathbb{N}}$ the observed time series and let $(\mathbf{X}_t)_{t\in\mathbb{Z}}$ be a homogeneous Markov chain defined by its state space $S = \{s_1, ..., s_N\}$, $N \in \mathbb{N}^\star$ and the $(N \times N)$ transition matrix $\mathbf{A}$ with $a_{ij} = P(\mathbf{X}_{t+1} = s_i \mid \mathbf{X}_t = s_j)$, $(i, j) = [1, ..., N]^2$. If we suppose, with no loss of generality, that the chain state space is the canonical basis of $\mathbb{R}^N$ and we note $\mathbf{v}_{t+1} = \mathbf{X}_{t+1} - E[\mathbf{X}_{t+1} | \mathbf{X}_t]$, then an autoregressive hidden Markov chain model has the following form:

$$\begin{cases} \mathbf{X}_{t+1} = \mathbf{A}\mathbf{X}_t + \mathbf{v}_{t+1} \\ Y_{t+1} = F_{\mathbf{X}_{t+1}}(\mathbf{Y}_{t,t-p+1}) + \sigma_{\mathbf{X}_{t+1}}\, \varepsilon_{t+1} \end{cases} \quad (1)$$

where $\mathbf{Y}_{t,t-p+1}$ defines the vector $(Y_{t-p+1}, ..., Y_t)$, $F_{\mathbf{X}_{t+1}} \in \{F_{s_1}, ..., F_{s_N}\}$ is an autoregressive function of order p, $\sigma_{\mathbf{X}_{t+1}} \in \{\sigma_{s_1}, ..., \sigma_{s_N}\}$ is a real strictly positive number, $(\varepsilon_t)_{t\in\mathbb{N}}$ are independently, identically distributed according to a standardized Gaussian law. In particular, we apply nonlinear autoregressive functions, such as the multilayer perceptrons, to consider the so-called hybrid HMC-MLP models (Hidden Markov Chain - Multilayer Perceptron models).

3 Research of a HMC-MLP Model

We focus on daily values of the WhIMS, computed from the CAC40 French stock market index, from July 9th, 1987 until April 30th, 2008 (5,243 observations). We first investigated whether the series (estimated using the Quasi-Maximum Likelihood method) is "regime switching" and what type of structure should be chosen and, secondly, if we can distinguish different market behaviors by using a hybrid Hidden Markov Chain – Multilayer Perceptron model (HMC-MLP).

The linear model and the one hidden-layer perceptron that were considered in a preliminary study showed that the significant lags were 1, 2, 4, 5 and 6. We fixed this input vector and we let both the number of experts and hidden units vary up to three. In the end, two configurations were selected and further investigated, on the basis of two empirical criteria: the first architecture is the one to have the smallest mean squared error and the second is the one with the "best" transition matrix. In the latter case, the "best" transition matrix is the one empirically generating the most stable segmentation of the series, that is its trace divided by its dimension is the closest to one.

3.1 Prediction Performance

First, we compare the estimated HMC-MLP model with the results of a linear ARMA and of a Multilayer Perceptron. After investigating all ARMA(p, q) models, for p and q in a specified range (both lower than 10), an AR(6) model minimizing the BIC criterion was selected. The research of the Multilayer Perceptron is done using the Baum-Welch algorithm [2] for Maximum Likelihood parameter estimation and the Viterbi algorithm [11] to compute the most probable sequence. The "best" model is chosen to minimize the Bayesian Information Criterion (BIC) and the "Statistical Stepwise" algorithm is used to eliminate the non-significant connections, once the "dominating" perceptron is found. The final selected model has two hidden units, twelve parameters and the input vector time dimension goes up to 6.

Comparisons of the mean squared error (MSE) suggest there is no special interest in considering a HMC-MLP model for forecasting the exact value of the WhIMS as there is no sensible improvement when compared to the linear model or to the perceptron (results available upon request). Intuitively, this negative result is not surprising when recalling that sudden and violent movements in the market are clearly difficult to predict, and that crises are often due to exogenous and unforecastable events (*e.g.* for the September 2001 terrorist attack). Nevertheless, a state representation of the turbulence is of interest since a qualitative assessment (crisis *versus* non-crisis period) is worthy and decisive for financial applications.

3.2 A Three-state HMC-MLP Model

The research of a three state model having the most significant segmentation of the series leads to select an architecture with the following estimated transition matrix:

$$\hat{A} = \begin{pmatrix} .91 & .06 & .03 \\ .02 & .83 & .15 \\ .02 & .13 & .85 \end{pmatrix}$$

The three experts are three hidden units perceptrons. The associated estimated variances are, respectively, .77, .36 and .24 and the mean squared error is 1.03. The interpretation of a three-state model is however not straightforward than a simple two-state one (crisis *versus* non-crisis), but preliminary results suggest that a 3-state model allow better discrimination of crisis and non-crisis periods than a 2-state model. We present hereafter a robust Kohonen [4] classification for describing the behavior of the Wavelet-heterogeneous Index of Market Shocks subject to the three experts. But, since during the learning process the outliers influence the model by moving the neurons toward the outliers, we use a robust version of the SOM [5].

We consider the WhIMS and the conditional probabilities related to the three perceptrons as input variables. The three probabilities come from the HMC-MLP model and thus correspond to a denoized estimate of the market conditions.

We first conduct a hierarchical classification performed on the map. If we aggregate the information contained in this Kohonen map, by cutting the classification tree and only keep three clusters, we note that the WhIMS values migrate through clusters, from small to high values; the first and the last cluster are homogeneous and correspond, respectively, to periods of calm and crisis respectively. The intermediate cluster - associated to medium WhIMS values and the third expert, but also to mixtures of experts mentioned above - is less homogeneous. The separation between high values of the WhIMS - associated with the first expert - and low values of the Index - linked to the predictions of the second expert - are quite insensitive to the number of clusters. Performing a Kohonen map analysis has here the major advantage that the cut between regimes is less arbitrary, and show that a clear separation can be made based on the WhIMS value and the value of conditional probabilities to be in some states.

Finally, we investigate the behavior of returns in the three identified clusters. Table 1 presents some performance measures of portfolios corresponding to each identified regime. Differences between the three clusters clearly reflect the various disturbance regimes from high return-low volatility (cluster 1) to low return-high volatility (cluster 3), with an intermediate state which is undetermined and corresponds to a wider range of WhIMS. The cumulated differences on the whole sample amount to large discrepancies between conditional performances. Moreover, the state separation based on the WhIMS leads to a better discrimination of market conditions than those based on the other financial disturbance measures.

The three portfolios derived from the WhIMS are shown on Figure 1: the series “State 1” (“State 2”, “State 3”) is built considering the benchmark returns when the period is classified in the first (second, third) cluster (of the three-category classification) and a zero return when classified elsewhere, either in the second (first, first) clusters or third (third, second). While the first state (first cluster) is clearly associated with low WhIMS and high returns, the second cluster is more in line with volatile markets, whilst the third cluster is generally associated with large drops in the market. These results suggest that jumps or regime switching in volatitily could potentially strongly affect portfolio allocation.

4 Conclusion

This paper proposes to model the WhIMS - an index of market disturbances - in a HMC-MLP framework to account for potential regime switching in financial turbulences. A non-linear classification, such a robust Kohonen map analysis, based on the WhIMS and conditional probabilities of the HMC-MLP model allows to identify and characterise stock market conditions. A major issue remains to determine statistically and no more economically the number of regimes in the HMM ([3] and [8]). From an asset allocation and risk-management perspective, a promising direction of future research would be to investigate how the identification of market condition regimes based on expert conditional probabilities altogether with WhIMS values affects the investor's portfolio optimisation

problem.

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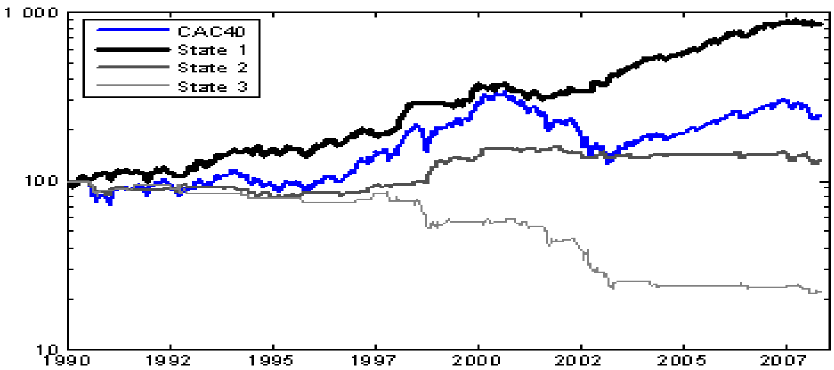
Fig. 1: Cumulated Return Series Related to the State of Turbulence of the CAC 40



Source: Bloomberg. Weekly CAC40 Index data between January 1st. 1987 and April 30th. 2008. "State 1" ("State 2". "State 3") corresponds to a series of cumulated returns (base 100 in January 1st. 1990) when the WhIMS and the conditional expert probabilities are classified in the first (second, third) cluster on a three-category string (semi-logarithmic scale). Computations by the authors.

Table 1: Comparison between Market Characterizations based on a HMM-MLP Modelling of the IMS, WhIMS, Implied Standard Deviation or One-year Volatility

	Freq.	Return	Vol.	Sharpe Ratio	Up	Large Down
All States						
CAC40	100	5.24	20.56	0.09	54.38	10
State 1						
WhIMS	63.68	25.5	15.5	1.42	60.54	4.09
IMS	44.54	20.85	15.58	1.11	59.12	4.14
VIX	35.78	11.36	17.66	0.45	55.76	7.58
Vol.	31.68	15.25	20.7	0.57	57.34	9.56
State 2						
WhIMS	29.84	-15.69	24.43	-0.79	44.93	17.75
IMS	35.03	-2.25	19.84	-0.29	51.85	12.04
VIX	37.08	7.99	15.51	0.29	57.14	6.71
Vol.	36.32	2.28	19.36	-0.06	51.94	7.46
State 3						
WhIMS	6.49	-47.44	35.49	-1.44	38.33	35
IMS	20.43	-11.58	29.26	-0.52	48.68	19.04
VIX	27.14	-5.71	28.62	-32.19	49	17.53
Vol.	32	-0.7	21.71	-19.32	54.39	13.18

Source: Bloomberg. Weekly CAC40 Index data between January 1st, 1987 and April 30th, 2008. Computations by the authors. The IMS corresponds to the Index of Market Shocks (Maillet and Michel, 2003), whilst the WhIMS to the Wavelet-heterogeneous Index of Market Shocks, the VIX the Index of Implied Volatility and the Volatility correspond to the one-year daily annualized volatility of returns on the CAC40. All figures - except Sharpe ratios - are expressed in percentages. The column "State" indicates the regime issued from the classification. Frequency represents the number of period in the state. Mean, volatility and Sharpe ratios represent annualized first and second central conventional moments of the conditional return in the various states. Up (Large down) frequency counts the number of positive (large negative) return in each state conditional samples.