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Zero Phase-Lag Synchronization through Short-Term Modulations

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Abstract. Considering coupled phase model oscillator systems with non-identical time delays, we study the possibility of close-to-zero phase-lag synchronization (ZPS) without frequency depression (FD). FD refers to nearly vanishing frequencies of the synchronized oscillators (in comparison to the intrinsic frequencies); its absence is crucial for interpretations related to brain dynamics. Discussing an extension of the Kuramoto model, it is demonstrated that ZPS without FD may arise by allowing for dynamical parameters. Two models are presented: one is based on short-term modulation of the delays, while the other assumes static delays but short-term modulation of coupling strengths. We also speculate on possible relevance of such mechanisms with respect to assembly formation by relating the frequency of the synchronized oscillation to recently proposed pattern frequency bands.

1 Introduction

A surprising feature of brain dynamics, not yet understood, is the observation that synchronization occurs with nearly zero phase-lag despite substantial time delays; consider as illuminating examples the experiments with visuomotor intergration task reported in [1] and [2, figure 2a]. The time delays result from delays due to synaptic transmission, post-synaptic integration, onset of the neural spiking, and transmission along the axon. As an indication for the size of delay times, one may consider onset latencies; see the review in [3].

Here, an analog problem is discussed in the context of coupled phase model oscillators. We demonstrate that allowing for short-term modulations of the system parameters may establish close-to-zero phase-lag synchronization (ZPS) without frequency depression (FD) (the meaning of FD gets obvious in the context of example 3). It is important to model ZPS through a dynamics that avoids FD, the reason being that the relevant rhythms in the brain are of relatively high frequencies given by the so-called gamma range (see [1, 2]).

In section 2, two models are introduced (models I and II) that demonstrate how ZPS without FD may be achieved if one allows for short-term modulations of parameters. Examples may be found in section 3. Section 4 contains a short discussion and an outlook.

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2 Synchronization of Phase Model Oscillators without Phase-Lags and Frequency Depression

2.1 Zero phase-lag synchronization through modulated time delays

Model I assumes that some underlying mechanisms realize an adaption of time delays such that the effective dynamics may be related to a system of the form given by

$$\text{model I } : \begin{cases} \lambda \dfrac{d\theta_n}{dt}(t) & = \lambda\omega_n + \dfrac{K}{N}\displaystyle\sum_{m=1}^{N} \sin_{nm}(\theta, \tau(t), t) \\ \lambda_\tau \dfrac{d\tau_{nm}}{dt}(t) & = f(\bar{\omega}\tau_{nm}) \ , \end{cases} \tag{1}$$

where the synchronizing terms with delays were abbreviated by

$$\sin_{nm}(\theta, \tau, t) = \sin(\theta_m(t - \tau_{nm}) - \theta_n(t)) \ . \tag{2}$$

Each oscillator n, $n = 1, ..., N$, is described in terms of the phase coordinate θ_n. The intrinsic frequencies are given by ω_n, λ is a time-scale, the coupling strength between the oscillators is given by $K > 0$, and the time delays are given by $N(N-1)$ values for $\tau_{mn} \geq 0$ ($\tau_{nn} = 0$). The λ_τ is another time-scale that should be chosen such that the modulations of the delays are short-term.

The $\bar{\omega}$ is chosen to describe an average frequency, $\bar{\omega} = (1/N)\sum_{m=1}^{N} \omega_m$; the $f(x)$ is chosen such that $f(x_0) = 0$ and $f'(x_0) < 0$ if and only if $x_0 = 2\pi N$, where N is integer. As an example, we consider $f(\bar{\omega}\tau_{nm}) = -\sin(\bar{\omega}\tau_{nm})$.

In the limit $\lambda_\tau \to \infty$, the delays are static and equation 1 reduces to the Kuramoto model with time delays. Its dynamics is illustrated with examples 1 to 3 in section 3 (example 3 shows FD).

In case of $\lambda_\tau < \infty$, the dynamics of equation 1 implies ZPS without FD. This may be understood by assuming that $\tau_{nm} = 2\pi N_{nm}/\bar{\omega}$, with N_{nm} some integer, and realizing that the trajectories $\theta_n = \bar{\omega}t + \phi + \epsilon_n$, $|\epsilon_n| \ll 1$, $n = 1, ..., N$ (that is, ZPS without FD), where ϕ is an arbitrary phase shift, are consistent with equation 1 (with K large enough). This argument is confirmed with example 4.

2.2 Zero phase-lag synchronization through modulated couplings

The following model, in contrast to the model of section 2.1, assumes that the time-delays are static; it results from modifying the Kuramoto model by assuming dynamical couplings that are modulated according to

$$\text{model II } : \begin{cases} \lambda \dfrac{d\theta_n}{dt}(t) & = \lambda\omega_n + \dfrac{K}{N}\displaystyle\sum_{m=1}^{N} \cos(\Omega(t)\tau_{nm}) \sin_{nm}(\theta, \tau, t) \\ \lambda_\Omega \dfrac{d\Omega}{dt}(t) & = -\Omega(t) + \bar{\omega} \ . \end{cases} \tag{3}$$

where $\lambda_\Omega > 0$ is another time scale that is to be chosen small enough so that the modulation of the coupling strength is short-term. In the limit $\lambda_\Omega \to 0$, the Ω dynamics could be eliminated from equation 3 after replacing $\Omega \to \bar{\omega}$ in the phase dynamics equation. We do, however, keep the dynamics of Ω in order to model the modulation starting from non-modulated couplings, that is, with example 5 we start from $\Omega(0) = 0$, giving $\cos(\Omega(0)\tau_{nm}) = 1$.

The reason why equations 3 may imply ZPS without FD is less obvious than with equations 1. In the following, we can only give a sketch of the reasoning. Consider a Kuramoto-like dynamics with a synchronizing term given by $\sin(\theta_n - \theta_m + \Omega\tau_{mn})$ (with time delays as in equation 2). On one hand, with $\theta_n = \bar{\omega}t + \phi$ (see section 2.1), this turns into $\sin(-\bar{\omega}\tau_{mn} + \Omega\tau_{mn})$ and a dynamics that implies $\Omega \to \bar{\omega}$ (as the one in equation 3) will let the sinus term approach zero. On the other hand, using a trigonometric identity, we find that

$$\sin(\theta_n - \theta_m + \Omega\tau_{mn}) = \sin(\Omega\tau_{mn})\cos(\theta_n - \theta_m) + \cos(\Omega\tau_{mn})\sin(\theta_n - \theta_m) \ . \quad (4)$$

It is then obvious that the phase dynamics of equation 3 is obtained from such a Kuramoto-like model if the intrinsic frequencies are (dynamically) shifted to eliminate the first term on the r.h.s. of equation 4. Given the fact that strong enough couplings K allow for shifting the intrinsic frequencies without destroying ZPS, it may then be understood that equation 3 implies ZPS (without FD, that is, $\theta_n = \bar{\omega}t + \phi + \epsilon_n$, $|\epsilon_n| \ll 1$, for $n = 1, ..., N$) as long as the coupling strength is strong enough to tolerate such shifts. The argued behavior of model II will be demonstrated with example 5.

3 Examples

The following examples 1 to 5 are based on a network with $N = 10$ units with intrinsic frequencies ω_n. The average value of these randomly distributed frequencies, drawn from a distribution with mean $\widehat{\omega}/2\pi = 40$ Hz, is given by

$$\frac{\bar{\omega}}{2\pi} = \frac{1}{2\pi N}\sum_{n=1}^{N}\omega_n = 39 \text{ Hz} \qquad \text{(examples 1-5)} \ ; \quad (5)$$

see figure 1A for an illustration of the corresponding time periods $2\pi/\omega_n$ with $2\pi/\bar{\omega} \simeq 26$ ms.

Except for example 3, the coupling strength is chosen to be $K = 2\lambda\widehat{\omega}$. (The λ may then be eliminated from equations 1 and 3.) The reason for this choice is that it turns out to be sufficiently large to establish synchrony in case of vanishing delays (example 1). Only example 3 uses another coupling strength, $K_{\text{strong}} = 2K$, to illustrate the frequency depressing effect of stronger couplings in case of the Kuramoto model with delays, that is, equation 1 with $\lambda_\tau \to \infty$ (this effect is not present in case of models I ($\lambda_\tau < \infty$) and II).

Examples 2 to 5 use non-vanishing time delays in a range between 20 and 100 ms; see the histogram of the randomly distributed time delays in figure 1B. These values may be compared to the time periods that correspond to the intrinsic time periods of figure 1A. The comparison shows that the time delays are substantial.

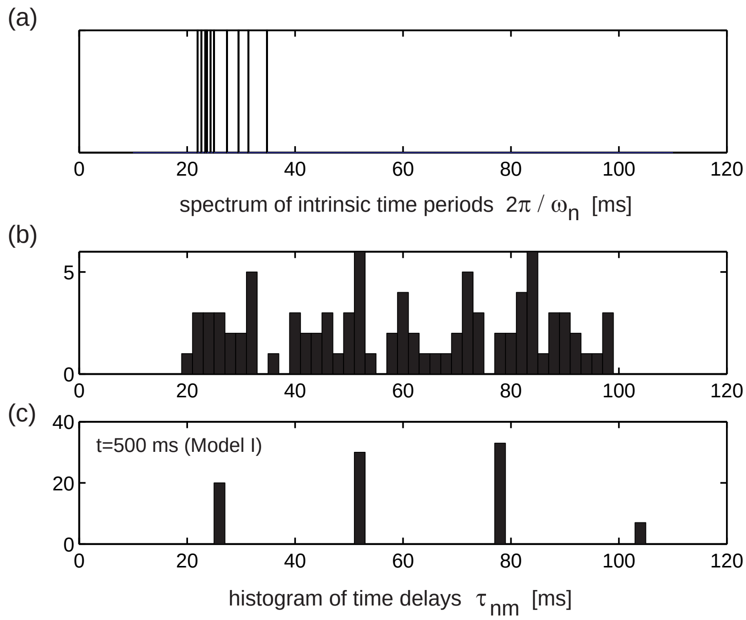

Fig. 1: (a) The spectrum of intrinsic time periods given by $2\pi/\omega_n$, $n = 1, ..., N$. (b) Histogram of the time delays τ_{nm} ($n \neq m$) (c) Example 4 (model I, where time delays are dynamical with initial values given by the values that are illustrated with (b)): Histogram of $\tau_{nm}(t)$ at $t = 500$ ms.

The examples use the same initial values and discretization is established through the Euler approximation with discrete time step $dt = 0.05\tau$.

Examples 1 to 3 illustrate the dynamics of the Kuramoto model, that is, the phase dynamics without using modulations of time delays or couplings (the Kuramoto model may be obtained from equation 1 as explained in section 2.1). We find that ZPS is established only at the expense of using strong couplings and allowing for FD; see figure 2A to C for examples 1 to 3, respectively. Example 4 (5) illustrates the dynamics of model I (II); see figure 2D (E). The results demonstrate that models I and II allow to establish ZPS without FD.

4 Discussion and Outlook

It may be of interest to point to a possible relevance with respect to pattern recognition and assembly formation as described in [5, 6]. There, patterns win a competition for coherence by taking a coherent state of a particular frequency (in a frequency band related to a pure pattern frequency [5]). Given such an approach, the mechanisms discussed here may be relevant, because they establish ZPS without FD for particular frequencies (given by $\bar{\omega}$). Thus, these mechanisms may serve to support the coherence of the winning patterns if the ω_n (and $\bar{\omega}$)

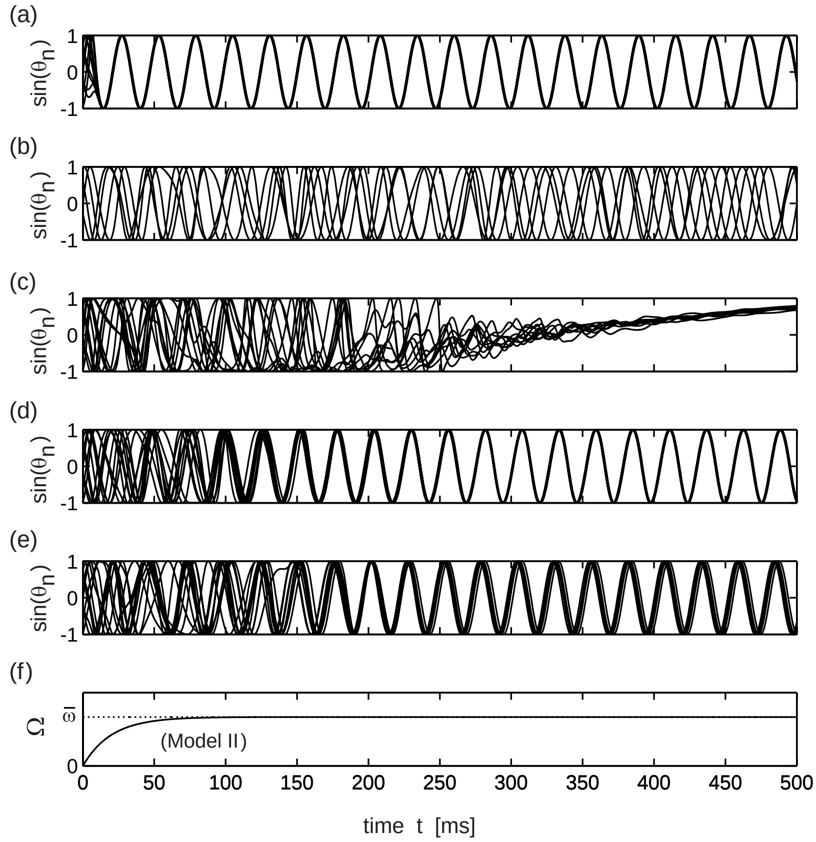

Fig. 2: (a-e) The dynamics of examples 1-5 is displayed by showing $\sin(\theta_n(t))$, $n = 1, ..., N$ ($N = 10$), for $t = 0$ to $t = 500$ ms. (a-c) Examples 1 to 3: These demonstrate the dynamics of the Kuramoto model, that is, no modulations of time delays or couplings are present. (a) Example 1: As a starting point, the synchronizing behavior is illustrated in case of vanishing time delays. (b) Example 2: In case of non-vanishing time delays, the ZPS may be destroyed (the panel shows only $n = 1, .., 4$). (c) Example 3: Frequency depression (FD). Stronger couplings (here, $K \rightarrow K_{\text{strong}} = 2K$) may realize ZPS at the expense of implying FD (FD refers to the resulting low value of the common frequency, it was first reported in the context of a two-dimensional lattice model [4]). (d) Example 4 and (e) example 5: the dynamics, resulting from equations 1 and 3, respectively, is displayed. Both models show ZPS without FD. In case of example 5 (model II), the corresponding modulation of Ω is shown in panel (f) (the dotted line gives $\bar{\omega}$).

are made dynamical in the sense described in [5, 6]. This would relate the $\bar{\omega}$ to pure pattern frequency bands of the winning patterns, while the other patterns (with different frequencies) may be desynchronized or frequency-depressed.

Corresponding to the brain dynamics origin of the problem, the approach of this paper, that is, to allow for dynamical parameters, may be reasonable, given the brain's capability of short-term modulations; see for example the review of short-term synaptic plasticity in [7]. Due to space restrictions, a more detailed study of the observed dynamics, as well as arguments for relating the mechanisms to brain dynamics must be left to future presentations. Notice, a complementary combination with another recent approach to enhancing ZPS given in [8] may be possible. With respect to brain dynamics, the main lesson to be learned is that some form of short-term modulations - not necessarily the ones presented here - may account for the surprising ZPS without FD of long-range and therefore strongly delayed connections between cortical units.

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