

A Performance Acceleration Algorithm of Spectral Unmixing via Subset Selection

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Abstract. An acceleration algorithm for spectral unmixing approach is proposed based on subset selection. The method classifies the pixels in a spectral image into accurate and approximated unmixing groups based on the similarity and dissimilarity of geomorphological features in neighboring areas. Real spectral images are used for unmixing benchmark tests for accuracy and performance verification. The results reveal good performance speedup with only small accuracy loss.

1 Introduction

In many applications, the spatial region covered by a pixel actually consists of a combination of known pure materials, so that the observed spectrum is a composition of the spectra of the materials present in the pixel. In Fig. 1(a), a high spatial resolution gray scale image of a section of core is presented, and the different shades of grey within the pixel indicate that it contains several different minerals. The spectrum corresponding to that pixel is shown in Fig. 1(b) [1].

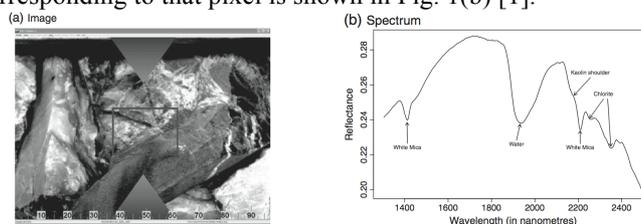


Fig. 1 Pixels consisting of several materials (a) image and (b) spectrum [1]

In recent years, the high spectral resolution and the increase of the spatial resolution attribute to modern hyperspectral sensors and new super-resolution techniques [2] pose new challenges that should be faced. Most of the traditional processing algorithms fail when the spectral resolution increases significantly [3]. As spectral mixture analysis is often implemented at a per-pixel or even sub-pixel level [4], it takes a long time to process the high-resolution information. A performance speedup algorithm is proposed to improve the very computationally intensive procedure with approximate endmembers. The pixels of an input spectral image are divided into an accurate group and an approximated group. Only pixels in the accurate group undergo fully computation for best-fitting mixture and the same mixture is applied to pixels in approximated group on assumption of spatial

smoothness. The proposed algorithm is verified by a spectral unmixing application based on subset selection for a comparative analysis of performance speedup and accuracy assessment.

A variable subset selection algorithm that uses a linear regression model for spectral mixture analysis is described in Section 2. A novel dirty tile algorithm with edge binarization and down-sampling approaches for pixel classification is presented in Section 3. The spectral unmixing experimental results including the approximation achieved and performance speedup are shown in Section 4.

2 Variable Subset Selection Algorithm for Linear Unmixing

In the application of spectral mixture analysis, a variable subset selection (VSS) algorithm [1] is used to estimate the land-cover fraction spatial spectra. We formalize the problem as follows. There is G pure spectra in the spectrum library, written as $\mathbf{X} \in \mathbb{R}^{D \times G}$, and a given spectrum $\mathbf{y} \in \mathbb{R}^D$ corresponding to a pixel in a spectral image. Both \mathbf{X} and \mathbf{y} contain D wavelengths. VSS searches for the best combination in terms of the least residual sum of squares (RSS) among all possible combinations:

$$\min_{|\mathcal{S}|=M} \|\mathbf{y} - \mathbf{X}_{\mathcal{S}}\boldsymbol{\beta}_{\mathcal{S}}\|_2^2 \quad (1)$$

where \mathcal{S} is a subset of variables (columns) from \mathbf{X} , $\mathbf{X}_{\mathcal{S}}$ is a submatrix retaining columns specified by \mathcal{S} , $\boldsymbol{\beta}_{\mathcal{S}}$ is the regression coefficient, $|\mathcal{S}|$ is the cardinality of set \mathcal{S} and $M \ll G$. The solution to (1) given $\mathbf{X}_{\mathcal{S}}$ is the left product of the Moore–Penrose pseudo inverse of $\mathbf{X}_{\mathcal{S}}$ and \mathbf{y} :

$$\boldsymbol{\beta}_{\mathcal{S}} = (\mathbf{X}_{\mathcal{S}}^T \mathbf{X}_{\mathcal{S}})^{-1} \mathbf{X}_{\mathcal{S}}^T \mathbf{y} \quad (2)$$

and the RSS associated with set \mathcal{S} , written as $\text{RSS}_{\mathcal{S}}$, is then

$$\text{RSS}_{\mathcal{S}} = \mathbf{y}^T \mathbf{y} - \mathbf{y}^T (\mathbf{X}_{\mathcal{S}}^T \mathbf{X}_{\mathcal{S}})^{-1} \mathbf{X}_{\mathcal{S}}^T \mathbf{y} \quad (3)$$

Although this linear regression model performs well, for every vector \mathbf{y} corresponding to a pixel in the spectral image, $\mathbf{X}_{\mathcal{S}}$ must be determined. The computational complexity of determining (1) is of the order of $O(DM^2 \binom{G}{M})$, which is a subset selection problem and it increases dramatically with M . However, if one can approximate $\mathbf{X}_{\mathcal{S}}$, namely $\mathbf{X}_{\tilde{\mathcal{S}}}$, then it is sufficient to compute (3) just once and the task of subset selection is avoided. This drastically reduces the computational complexity although the selected variables will not be the same as those obtained from performing full subset selection on each pixel. Due to spatial smoothness, pixels in a small neighborhood are very likely to share the same composition. Therefore it is reasonable to compute $\mathbf{X}_{\mathcal{S}}$ using VSS just once for a pixel in that neighborhood and then use it as an approximation $\mathbf{X}_{\tilde{\mathcal{S}}}$ for others. These neighboring pixels are called the approximated unmixing group. The difference between using $\mathbf{X}_{\tilde{\mathcal{S}}}$ and its true subset based on VSS is defined as follows, which we call the accuracy of approximation, or simply accuracy α :

$$\alpha = \frac{|\{\mathcal{T}: \text{RSS}_{\mathcal{T}} > \text{RSS}_{\tilde{\mathcal{S}}}\}|}{N_{|\tilde{\mathcal{S}}|}} \quad (4)$$

where \mathcal{T} is any subset of size $|\tilde{\mathcal{S}}|$ and $N_{|\tilde{\mathcal{S}}|}$ is the number of subsets with size $|\tilde{\mathcal{S}}|$, and α varies from 0 to 1.

3 Acceleration Approaches

The proposed novel accelerating approaches are based on the fact that in a spectral image, pixels from homogeneous and neighboring regions probably contain the same spectral mixture, while pixels from heterogeneous and distant regions are likely to be formed with different mixture. The pipeline of clustering algorithm is shown in Fig. 2.

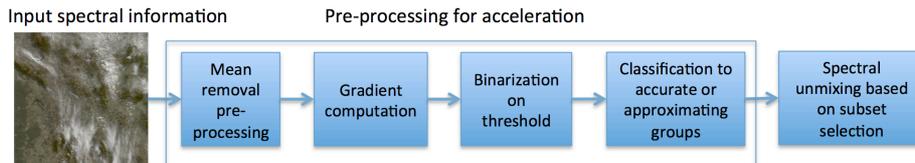


Fig. 2 Workflow of acceleration based on pixel grouping

3.1 Edge Binarization

In spatial image, the combinations of pure materials vary little in homogeneous regions such as a scene with the same land-cover. The boundary and inner “noisy” pixels that can be considered as edges, usually consist of different linear spectral combinations and sometimes cannot be simply classified to a single adjacent homogeneous region; therefore, those pixels require accurate spectral unmixing processing.

Mean removal algorithm is applied to reduce brightness variation before further processing for edge detection:

$$\begin{bmatrix} y'_1 \\ y'_2 \\ \vdots \\ y'_n \end{bmatrix} = \frac{1}{\sqrt{y_1^2 + y_2^2 + \dots + y_n^2}} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad (5)$$

As the Sobel operator is able to detect discontinuities in the gradient, it is applied to all the bands of spectral information with equal weights, and the gradient magnitude is then computed as:

$$|\nabla L(i)| = \sqrt{\sum_{i=1}^n L_x(i)^2 + L_y(i)^2}$$

$$L_x(i) = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} L(i), \quad L_y(i) = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} L(i) \quad (6)$$

where D is the number of bands in the spectrum. The edge pixels can be marked out by image binarization $f_{\lambda}(i)$:

$$f_{\lambda}(i) = \begin{cases} 1, S_i \geq (1 - \lambda/100)/n \sum_{j=1}^n S_j \\ 0, S_i < (1 - \lambda/100)/n \sum_{j=1}^n S_j \end{cases} \quad (\lambda : 0 < \lambda < 100) \quad (7)$$

S is the multi-band magnitude for each pixel with the Sobel edge detector. λ is the percentage of edge pixels with value 1. A larger λ results in more pixels classified to accurate unmixing processing and less performance speedup. The edge pixels will all be selected to perform best-fitting mixture computation in (2).

3.2 Dirty Tile

After edge binarization we consider a tile, which is a small subimage of the input spectral image, to be a “clean” tile when it contains edge pixels within a threshold; otherwise, it is a “dirty” tile. Every pixel in a dirty tile will be processed with full subset selection to search for the best-fitting combination.

Given μ the percentage of edge pixels in a whole spectral image, tile size T and the edge binarization result $f_\lambda(i)$, the tiles are classified to either dirty tile $f_T(k) = 1$ or clean tile $f_T(k) = 0$ as follows:

$$f_T(k) = \begin{cases} 1, & \sum_{i=1}^{T \times T} f_\lambda(i) \geq T * T * \mu \\ 0, & \sum_{i=1}^{T \times T} f_\lambda(i) < T * T * \mu \end{cases} \quad (8)$$

where T it is another important parameter for acceleration and accuracy. A tile with the same μ but larger T often results in fewer dirty tiles and therefore less computational workload, though this is in general less accurate in terms of finding the right combinations.

The tile-based accelerating approach is based on the idea that when a tile consists of many edge pixels, it is likely that pixels surrounding these edges also tend to consist of a mixture different from either the edge pixel or a single adjacent homogenous area, therefore the whole tile goes through accurate computation rather than approximation. As a result, edge regions actually get widened to obtain overall accuracy improvement over the whole input spectral image.

3.3 Down-sampling

In high-resolution spectral data, a continuous landscape is usually covered by many tiles, and not all the clean tiles have exactly the same linear combination of pure materials, thus downsampling is used to select representative pixels in clean tiles. In a clean tile. The representing pixel performs full unmixing procedure and the rest pixels, except the edge pixels, use (3) to compute proportions of endmembers with \mathbf{X}_g of the representing pixel. In this way, the very computationally intensive procedure in (2) is saved for most of the pixels in a clean tile.

4 Evaluation

The acceleration algorithms and the corresponding accuracy gains were verified on 16 groups of 512x512 real high-resolution spatial images containing different geomorphological features. The pseudo-color images and processed images at each stage are shown in Fig. 3.

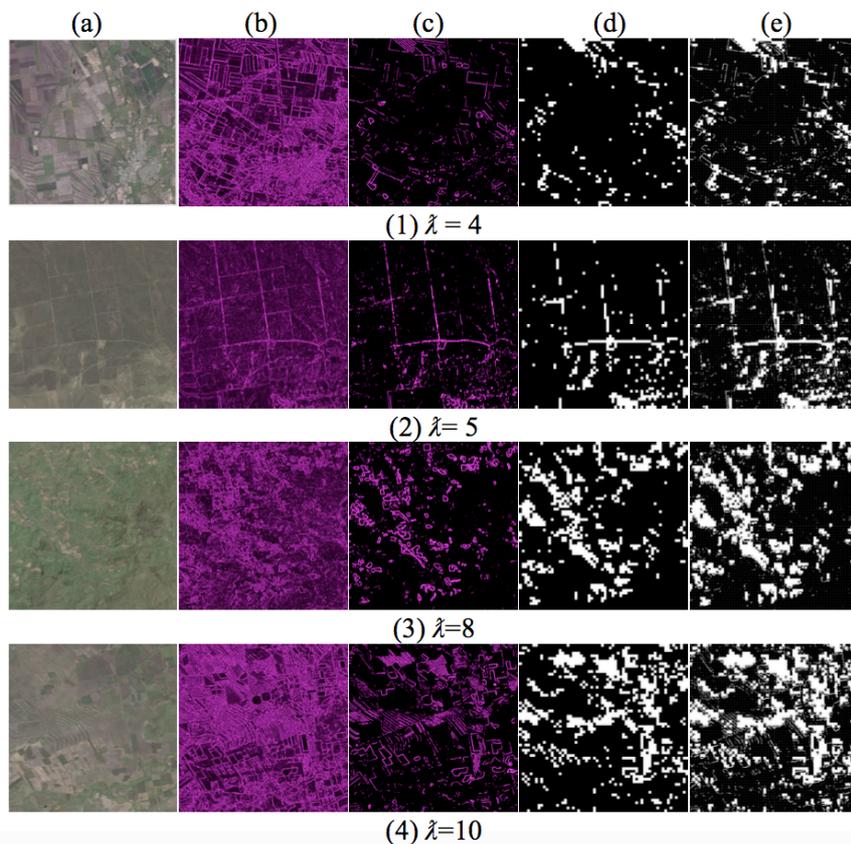


Fig. 3 Accelerating pipelines: (a) pseudo-color images (b) edge pixels with Sobel operator on normalized spectral vectors (c) edge binarization results at different λ (d) dirty tiles with $\mu=0.2$ (e) pixels of fully unmixing computation with tile size $T=8$

The performance speed up and approximation profile results are shown in Fig. 4, where the binarization parameter λ is valued 1, 2, 5, 8, 10 on the points of the curves from left to right. Larger values of T and λ will only be suitable for higher resolution images. Otherwise the overall accuracy will decrease.

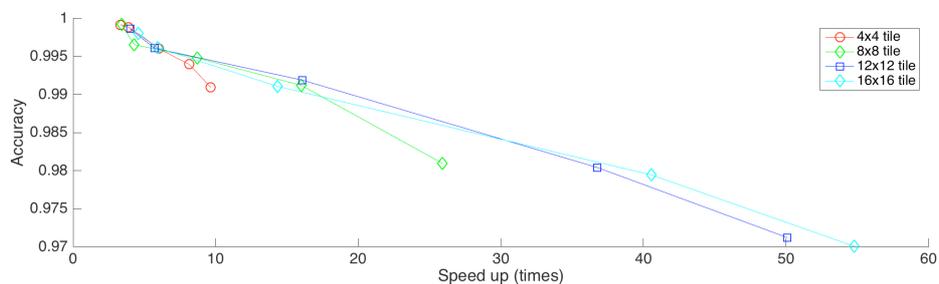


Fig. 4 Speedup versus accuracy for varying λ and T

We find that when λ is small, the speedup is mainly decided by the representation sampling frequency that is relevant to tile size. When λ is increasing, the speedup will decrease. However, as more edge pixels and dirty tiles are involved in full computation, the overall accuracy is improved. The distribution of the deviations between best-fitting and estimated endmembers is shown in Fig. 5.

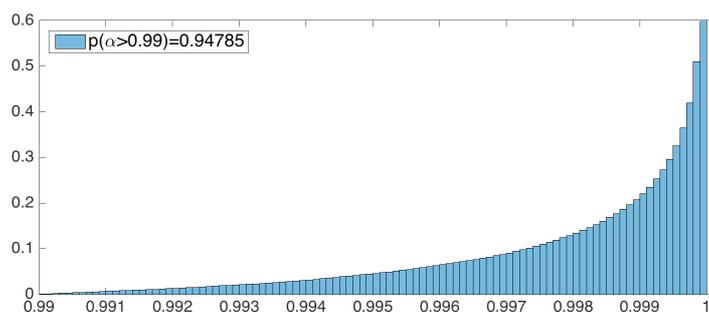


Fig. 5 Deviation distribution on $T=4$ and $\lambda=2$

Conclusion

This paper introduces a novel approach to accelerate the mixture analysis of high-resolution spatial spectral information by pixel classification with optimized subset selection algorithm. The proposed edge binarization and dirty-tile algorithms significantly reduce the most time-consuming workload in searching for the best linear combination.

Compared to down-sampling methods, these acceleration approaches group more best-fitting pixels around boundary pixels and the outlier pixels inside homogenous areas. The speedup and accuracy might vary due to different surface materials, resolution and spectral library, but the approach provides an efficient and fast procedure for spectral unmixing analysis. Many similar image-processing applications can be pre-processed with this method for faster and more efficient analysis.

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