

very moderately with $m = n$, because our algorithms spend a large fraction of their time on the projection step, which does not depend on m , neither n .

4 Conclusion and discussion

We have shown that extending NMF to handle polynomial signals directly enables recovery of smoother features and is less sensitive to noise than standard HALS applied to discretized signals.

We have presented two new algorithms to perform NMF with polynomial signals, and showed that they provide accurate results. Another advantage compared to existing methods is that their computation times only increase moderately with the problem size, making them more interesting to deal with large-scale problems (i.e. with large numbers of observations or discretization points) compared to the previously proposed least-squares approach from [8].

Our algorithms are naturally well-suited to deal with data originating from nonnegative polynomials. Adapting the methods to other nonnegative interpolating functions, such as splines, would be an interesting topic for future research.

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